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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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CONTENTS

NUMBER I

NUMBER II

NUMBER III

NUMBER IV

NUMBER V

THE

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AND ASTRONOMICAL PHYSICS

VOLUME LII — JULY 1920 — NUMBER 1

EXCITATION OF THE SPECTRUM OF HELIUM

By K. T. COMPTON and E. G. LILLY

ABSTRACT

Minimum voltage for excitation of the helium spectrum.—The spectrum was excited by bombarding very pure helium with electrons from a hot filament cathode at various pressures up to 24 mm. The *singlet and doublet series and band spectrum* appeared simultaneously at 25.5 volts with low pressures and at voltages as low as 20 with higher pressures and current-densities. The corresponding figures for the *enhanced line* λ 4686 are 80 and 55 volts. These results indicate that the minimum voltages for the excitation of a normal helium atom by a single electronic impact are 25.5 and 80 volts; but when multiple impacts assist, the corresponding voltages may drop to 20 and 55, but no lower. This last value agrees with that predicted by the Bohr theory for the energy necessary to displace the second electron from a singly ionized helium atom. The fact that after striking the arc it could be maintained on as little as 8 volts by using high currents indicates that in an intense discharge a large proportion of the atoms are in an abnormal state and therefore require less energy for excitation.

Helium arc spectrum; relative intensity of series and bands.—As the voltage was increased, the sharp subordinate series became relatively weaker. As the pressure was increased, the band spectrum became stronger and the enhanced line weaker. The band spectrum was stronger near the cathode, while the enhanced line was stronger near the anode.

Spectrum of neon in helium.—A trace of neon in the helium arc caused the neon lines λ 5331 and λ 5431 to appear with remarkable intensity.

INTRODUCTION

Knowledge of the conditions determining the excitation of the helium spectrum is of particular interest because of the possibility which it affords of testing the recently proposed theories of atomic

structure. An investigation of the Doppler effect led Stark[1] to the conclusion that the parhelium (singlet) series are due to electronic disturbances in helium atoms which have lost two electrons, while the helium (doublet) series are due to disturbances in the singly charged atoms. Presumably such disturbances must be ascribed to recombinations with electrons, since the doubly charged atoms are devoid of radiating electrons, so that the actual radiating systems in the two types of spectra are singly charged and neutral atoms, respectively. Rau,[2] on the other hand, showed that both types of series appear simultaneously under conditions in which no double ionization is possible, so that probably both are due to radiation from neutral atoms under different modes of disturbance.

The enhanced lines, of which the most prominent is the line λ 4686, have been assumed to be due to excitation of singly charged atoms, or to the return of one electron to doubly charged atoms. This has been verified by Rau's observation that the line λ 4686 required for excitation a potential difference of at least 80 volts between electrodes, whereas the ordinary line spectrum was excited at voltages as low as 25 volts under favorable conditions.

Bohr's model of the helium atom predicts single ionization of the helium atom by electrons which have fallen through 28.7 volts, and double ionization at 83 volts. It predicts the spectral series of the enhanced group, i.e., of the ionized atoms, but not those of the ordinary series belonging to the neutral atoms. The theory is quite definite as regards the energy, equivalent to 54.3 volts, necessary to remove the second electron from an atom which has already lost one electron, but the energy required to remove the first electron and the spectral radiation due to its complete or partial removal are uncertain because of lack of knowledge regarding the behavior of the second electron while the first is being removed.

Experiments have shown that the first ionization is at about 25.5 volts, and the first stage in this displacement requires about

[1] J. Stark, A. Fisher, and H. Kirschbaum, *Annalen der Physik*, **40**, 499, 1913.

[2] *Sitz. Ber. der Phys. Med. Ges. zu Würzburg*, p. 20, 1914.

20 volts.[1] This suggests an emission and absorption series with the first member about 605 A and converging at about 484 A. No such series has been observed. Taking 54.3 volts as the additional energy required to remove the second electron, we have 79.8 volts as the potential difference necessary to doubly ionize an atom by a single impact. Thus the ordinary series of helium should be excited at any voltage above 25.5 and the enhanced series at any voltage above 79.8, *provided the excitation is due to single impacts against normal atoms by electrons which have gained energy through the entire potential drop.*

In an intense discharge, however, there are present many atoms which are already entirely or partially ionized by previous impacts or by the absorption of radiation from previous impacts with neighboring atoms.[2] In such cases it is also possible to place theoretical lower limits to the voltage at which the series may be excited. Apparently 20 volts is a definite minimum voltage for the excitation of radiation of any sort from helium, since the displacement corresponding to this voltage is prerequisite to any further displacement of the radiating electron, and it is known that a normal helium atom can absorb no energy from impacting electrons which have fallen through less than 20 volts. But with an intense bombarding stream of 20-volt electrons there will be present atoms in all stages of single ionization. The minimum voltage for double ionization in such a case is 54.3 volts, if we except the extremely improbable case of double ionization by successive impacts against atoms which are already singly ionized.

We should therefore expect the excitation of the helium spectrum by electron impacts to be subject to the following conditions:

a) In gas at high pressure, the ordinary series lines should be excited at voltages above 25.5 volts if the electron stream is not too dense, or above 20 volts if the bombarding stream is very dense. In no case should radiation be excited at less than 20 volts. The enhanced series should not be observed, owing to the

[1] F. Horton and A. Davies, *Roy. Soc. Proc.* A., **95**, 408, 1919; and *Phil. Mag.* (6), **39**, 592, 1920; J. Franck and P. Knipping, *Physikalische Zeitschrift*, **20**, 481, 1919; K. T. Compton, *Phil. Mag.* (in print).

[2] K. T. Compton, *Physical Review* (in print).

small chance of an electron falling through the requisite higher voltage without losing its energy in the production of ordinary radiation at an intervening impact.

b) At low pressures, where the mean free path of an electron is comparable with the distance between the electrodes, the ordinary series should appear at 25.5 volts or more, and the enhanced series at 79.8 volts or more.

c) At intermediate pressures, with a very intense bombarding electron stream, the ordinary spectrum should appear at 20 volts and the enhanced series at 54.3 volts.

The following experiments entirely verify these predictions.

EXPERIMENTAL RESULTS

Radiation was excited by a stream of electrons emitted by an incandescent tungsten wire and drawn through the gas to a nickel disk anode by a field which could be varied at will between 0 and 120 volts by means of series and shunt resistances. Various straight and coiled filaments were used, whose diameters ranged between 0.06 mm and 0.25 mm, and whose lengths were from 1 cm to 4 cm in different cases. These electrodes were sealed in a glass tube of 5 cm diameter, which was thoroughly "baked out" in an electric heater at 350° C. for several days, in order to drive out water-vapor. Carefully purified helium was introduced from a reservoir through traps and cocoanut charcoal tubes immersed in liquid air, and the gas pressure could be regulated between 0 and 24 mm by a hand mercury pump between the tube and the reservoir. Another large tube of cocoanut charcoal was sealed directly to the experimental tube. During operation a cooling air blast was directed against the tube to prevent excessive heating, with the resulting liberation of water-vapor. With these precautions a very intense helium arc could be obtained which showed beautifully the line spectrum and the complicated band spectrum recently discovered by W. E. Curtis[1] and A. Fowler[2] but without any detectable trace of hydrogen, mercury vapor, neon, or other impurity. The most satisfactory distance between the electrodes was found to be about 1.5 cm.

[1] *Roy. Soc. Proc.* A., **89**, 146, 1913. [2] *Ibid.*, **91**, 208, 1915.

With this apparatus the predictions (*a*), (*b*), and (*c*) above were verified. With moderate electron currents the arc struck, with the simultaneous flashing out of the spectrum, at voltages generally between 25 and 35 volts. With the most intense currents, the filament being heated nearly to its melting point, the arc struck at 20 volts, but never below. After striking, however, it could be maintained on voltages as low as 8 volts. This indicated that the electronic bombardment and radiation density in the gas were so intense that practically all the atoms were in a partially ionized condition, the electrons returning toward their normal stable configuration being re-ejected before reaching it. Gas pressures between 3 and 10 mm were favorable for the production of the arc at this abnormally low voltage.

The enhanced line λ 4686 was observed to come in very strongly at 80 volts at low pressure and current-density. In the most intense arc it was observed at voltages as low as 55 volts, but always with increased intensity at 80 volts. It was not observed at gas pressures above 10 mm.

The helium band spectrum, consisting of more than 1300 lines in the visible spectrum alone, is of great interest because the theories of the structure of helium have suggested no explanation of it. Curtis allows some uncertainty regarding the origin of these lines, since hydrogen was present in his experiments. In the present experiments, however, hydrogen was entirely absent, while the brightest of the band lines could be made very intense indeed, comparable in intensity, for example, with the sharp subordinate parhelium line λ 5048. Simply by the use of a direct-reading Hilger wave-length spectrometer we were able to identify more than 250 of these lines, which agreed exactly with those found on the plates published by Curtis for the same spectral region. Inasmuch as he is working out the accurate values of the wave-lengths, we made no effort to do this, but devoted our attention to a study of the conditions of their excitation.

High gas pressure is favorable to the excitation of the band spectrum. It was relatively most intense at the highest pressure used, i.e., 24 mm, while it was barely visible at 3 mm and not seen at all at 2 mm. As nearly as could be judged, the voltage necessary

to excite the band spectrum is identical with that necessary to excite the ordinary line spectrum. The band spectrum was relatively most intense near the cathode, while the enhanced spectrum was only visible near the anode if the voltage was not much above the minimum exciting voltage. This suggests that the enhanced spectrum is entirely or chiefly excited by electron impacts, while the band spectrum is excited chiefly by impacts by positively charged atoms.

TABLE I

Line	Series*	Exciting Voltage							
		30	42	50	62	75	85	95	110
7066.....	(1p†)−(ms)	0.3	0.3	0.2	0.2	0.2	0.1	0.0	0.0
6678.....	(1P) −(mD)	0.7	0.7	0.7	0.7	0.8	0.7	0.7	0.6
5876.....	(1p) −(md)	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
5048.....	(1P) −(mS)	0.5	0.5	0.4	0.4	0.4	0.3	0.3	0.3
5016.....	(1S) −(mP)	0.8	0.9	0.8	0.8	0.9	0.8	0.9	1.0
4922.....	(1P) −(mD)	0.5	0.6	0.5	0.6	0.6	0.6	0.6	0.6
4713.....	(1p) −(ms)	0.6	0.5	0.6	0.6	0.6	0.5	0.4	0.3
4686.....	enhanced	0.0	0.0	0.0	0.0	0.0	0.05	0.1	0.1
4472.....	(1p) −(md)	0.7	0.8	0.7	0.8	0.8	0.7	0.8	0.7
4438.....	(1P) −(mS)	0.1	0.1	0.1	0.1	0.05	0.05	0.0	0.0
4388.....	(1P) −(mD)	0.3	0.3	0.2	0.3	0.2	0.2	0.1	0.1

* For notation see F. A. Saunders, *Astrophysical Journal*, **50**, 151, 1919.
† p=1 mm.

Very careful tests were made in an effort to distinguish small differences in the voltage necessary to excite different lines in the same series or corresponding lines in different series. With the exception of the enhanced lines, however, no difference could be detected. In general the most intense lines are seen at slightly lower voltages than the others, but tests under various conditions of excitation as well as photographic tests with different times of exposure have shown that, if any difference exists, it is extremely small. This is to be expected, since the first step in excitation, at 20 volts, requires a voltage so much higher than any succeeding step that all others may occur by direct or indirect successive impact after the first has been accomplished.

We verified Rau's observation of the relative decrease in the intensities of the lines of the sharp subordinate series with increasing

voltage. The foregoing table gives an example of the change of intensities of some of the lines of the spectrum with change of voltage. In each case the intensities were estimated visually, and are given as fractions of that of the D_3 line. In general the lines $\lambda\lambda$ 7066 and 4713 of the sharp subordinate series of helium decreased in intensity with increasing voltage rather more rapidly than indicated in the above particular example.

We found no lines of any of the combination series. This is not surprising, since these lines are known to appear in regions where the electric field is very strong, and the field in the present type of light source is very much weaker than in the ordinary spark or Geissler tube discharge.

Finally, it may be desirable to record an interesting observation made in several experiments in which a slight trace of neon was mixed with the helium. Although this trace was so slight that only about a half-dozen of the neon lines could be seen, the neon lines near $\lambda\lambda$ 5331 and 5431 were quite bright, as bright as any except the four or five brightest lines of helium. Yet there are sixteen other lines given by Paschen[1] as one to four times as intense, which were either entirely invisible or so faint as to make their identification uncertain.

Palmer Physical Laboratory
Princeton, New Jersey
June 16, 1920

[1] *Annalen der Physik*, **60**, 405, 1919.

THE COLOR OF THE NEBULOUS STARS[1]

By FREDERICK H. SEARES and EDWIN P. HUBBLE

ABSTRACT

Color of forty-seven nebulous stars.—The ratio of the times of exposure with and without a yellow filter, producing images of the same size on an isochromatic plate, varies regularly with the spectral type for normal stars of a given absolute magnitude. Most nebulous stars, however, as the results in this paper show, are abnormally red for their spectral type. The color-indices of thirty-six out of the forty-two nebulous stars for which complete data are available are from 0.4 to about 3 color-classes in advance of their respective spectral types. The average *color excess* is about one color-class, which is five times the probable error. Two methods were used. The color-indices obtained from the *blue-yellow exposure ratios* determined with the 60-inch reflector were checked in part by those obtained from *polar comparisons of photographic and photo-visual magnitudes* made with the 10-inch Cooke refractor. The stars investigated are in R.A. 6^h to 7^h and 16^h to 18^h and include among others ρ Ophiuchi, σ Scorpii, 22 Scorpii, N.G.C. 1514, 2170, 2175, 2182, 2245, 2247, 2282, 2283, I.C. 444, 446. Several possible *explanations* of these results are discussed.

Suspected variable star.—The behavior of B.D.$+10°1771$, 8^M0, spectrum K0, suggests that it may be a variable.

Spectrum of nebulosity around nebulous stars N.G.C. 2245, 2247, 2282, I.C. 446, and ρ Ophiuchi is continuous, whereas the nebulosity around N.G.C. 1514 is gaseous.

In a paper presented to the Royal Society in 1791,[2] Sir William Herschel described a class of objects that he called "nebulous stars, properly so-called." These objects are found in or near the Galaxy, and are characterized by the fact that visually they present nothing to differentiate them from ordinary stars, except a surrounding mass of nebulosity; they are perfectly sharp and show none of the diffuseness found in those extra-galactic objects to which the name "nebulous star" is often applied. The nebulosity itself is frequently inconspicuous, and in some cases can be detected only on photographs of long exposure.

Historically, the nebulous stars of Herschel are of great interest, for it was through them that he first became convinced of the existence of luminous matter widely scattered throughout space, in a form quite different from that which constitutes the stars, and

[1] *Contributions from the Mount Wilson Observatory*, No. 187.

[2] *Phil. Trans.*, **81**, 71, 1791; *Collected Scientific Papers*, Vol. I, 415 (London, 1912).

was led to generalizations that have long dominated the field of stellar evolution. Their important contribution dates back a hundred and thirty years, but the interest which attaches to nebulous stars is not wholly historical, as will appear from the following.

The observations described below have grown out of a remark by Hubble that visually the central star of N.G.C. 1514 and of one or two similar objects are appreciably redder than their spectral types would indicate them to be.[1] Measures of the color of N.G.C. 1514 and 2245 by the method of exposure-ratios confirmed this suspicion. An examination of the color-indices available for certain other nebulous stars, although inconclusive, was suggestive that a real excess of color might be associated with these objects as a class. Systematic observations were then undertaken and sufficient data have been accumulated to put the matter largely beyond doubt. For the great majority of the objects observed there seems to be no doubt whatever; two or three stars appear to be exceptional, and two or three others included in the program are open to question, because their intimate association with nebulosity is not yet fully established.

Herschel's original list[2] of nebulous stars included eighteen. The number at present known is approximately one hundred; many of the later discoveries are due to Professor Barnard, to whom we are much indebted for unpublished data. The stars thus far observed, about fifty in all, occur in two groups, in R.A. 6^h–7^h and 16^h–18^h, and are mainly in southern declinations ranging from the equator to $-24°$. Their spectra are mostly of type B; a very few seem to be early A's; one is Oe5 and one G5.

The observations have been made with the 60-inch reflector by the method of exposure-ratios,[3] controlled in part by photographic polar comparisons with the 10-inch Cooke refractor. To reduce the measurement to a differential procedure, all the nebulous stars in the same part of the sky were photographed on the same

[1] Russell (*Proceedings of the National Academy of Sciences*, **5**, 398, 1919) has called attention to the abnormal color-indices of ζ, o, and ξ Persei, all situated in a region full of diffuse nebulosity.

[2] *Loc. cit.*

[3] Seares, *Proceedings of the National Academy of Sciences*, **2**, 521, 1916; **3**, 29, 1917; **5**, 232, 1919.

plate, along with a half-dozen other stars of known spectrum, not associated with nebulosity, whose colors are assumed to be normal. As a further aid in the elimination of systematic errors, the check stars were observed at approximately the same mean zenith distance as the nebulous stars.

Three stars of the group in 6^h–7^h were already out of reach toward the west when the observations were begun. Of this group we have observed thirteen nebulous stars and six check stars. For the second group the numbers are thirty-one and seven, respectively, although several of the nebulous stars occur on a single plate. The polar comparison plates supply results for two additional nebulous stars in the second group.

TABLE I

OBSERVATIONS OF NEBULOUS STARS

Exposure Ratio, 60-inch Reflector			Polar Comparisons, 10-inch Refractor
1920, Feb. 16, Plate	5209, 5210		1920, Apr. 12, Plate 5, Iso, 10^m
Apr. 21	5241		6 S30, 1
22	5244, 5250		7 S30, 1
23	5254		8 Iso, 10
24	5266		Iso plates used with filter D

To determine the ratio of exposures necessary to produce blue and yellow images of equal intensity, exposures, usually of 2, 4, and 8 seconds, are made on a Cramer Instantaneous Iso plate, thus giving a series of "blue" images. The yellow images are produced by exposing the same plate behind a yellow filter—32 and 64 seconds for B and A stars, 16 and 32 seconds for F and early G stars, and 8 and 16 seconds for later G's and for K and M stars. To identify the images additional 2-second exposures are made, and the order of one or both of the series is inverted. The blue images serve as a scale with which the exposure required to produce a blue image equal to a given yellow image may be read directly from the plate. The logarithm of the exposure-ratio ($\log E$) is used as the measure of color.

Since the relative size of the blue and yellow images depends upon their actual size, it is important that the images be kept within certain narrow limits. To this end each star photographed

is reduced to a standard apparent magnitude—usually the tenth, with a possible range of half a magnitude in either direction—by means of diaphragms and screens. In refined work the remaining small deviations may be compensated by slight corrections, although it is doubtful if anything is thereby gained, since the corrections are of the order of the unavoidable uncertainties. The exposure-ratio also depends upon the emulsion and, indeed, to some extent upon the individual plate.

The relation of $\log E$ to spectral type and Mount Wilson color-indices (color system of the 60-inch reflector) for a provisional mean curve is shown in Table II. The values refer to stars of zero absolute magnitude. For dwarf stars quite another series must be used,[1] but this is not needed for the present discussion. In the region of the B stars the curve is possibly a little too steep; but this cannot appreciably affect the present results.

TABLE II

SPECTRUM, EXPOSURE-RATIO, AND COLOR-INDEX

(Blue-Yellow, Provisional Values for $M = 0$)

Sp.	log *E*	C.I.	Sp.	log *E*	C.I.
B0	8.80	−0.40	G0	9.26	+0.80
B5	8.86	−0.20	G5	9.35	+1.00
A0	8.94	0.00	K0	9.45	+1.20
A5	9.02	+0.20	K5	9.55	+1.40
F0	9.10	+0.40	Ma	9.55	+1.40
F5	9.18	+0.60	Mb	9.55	+1.40

The effect of atmospheric extinction and fluctuations from plate to plate in the relative blue-yellow sensitiveness is to displace vertically the curve of $\log E$ and the spectrum. These and other similar factors, such as abnormal values of the coefficients of reflection from the mirror, all combine to form a plate constant whose value is determined by comparing the observed values of $\log E$ for the check stars with the normal curve. The application of the plate constant to the observed values of $\log E$ for the stars of unknown color reduces them to the curve from which the color-class[2] or color-index can be determined.

[1] Seares, *Proceedings of the National Academy of Sciences*, **5**, 232, 1919.

[2] Seares, *op. cit.*, **1**, 481, 1915.

Table III gives the values of the plate constants derived from the individual check stars and the adopted means. The check stars are all giants, but several of them deviate appreciably from zero absolute magnitude, to which the values of Table II refer. Allowance for this fact has been made with the aid of the curves in the *Proceedings of the National Academy of Sciences* (**5**, 232, 1919, Figs. 2 and 3). The results in Table III are affected both by the errors of measurement and the uncertainties in the spectral classification. Their combined influence upon a single value of the plate constant averages ±0.041, or ±0.10 mag. The average uncertainty for a plate is therefore ±0.017, or ±0.04 mag.

TABLE III

CHECK STARS AND PLATE CONSTANTS

(Unit = 0.01 in log E)

Star	Sp.	5241	5244	5254	Star	Sp.	5250	5266
Boss 1751.....	B8	− 4	−13	+ 1	Boss 3758....	F3	− 5	+ 5
1788.....	B8	− 3	−10	− 3	3810....	K0	−16	− 5
1944.....	B8	− 4	−13	−19	4137....	F9	− 9	−11
2008.....	F3	......	− 6	− 5	4193....	Ma	− 6	−11
2020.....	Mb	− 6	− 5	−11	4731....	F3	− 4	+ 4
2330.....	B3	−10	−14	− 6	5003....	B0	− 6	−12
....	......	......	......	......	5062....	A4	− 1	......
Means.......	......	− 5	−10	− 7	Means.....	......	− 7	− 5

Freed from plate error, the results for all the stars observed are in Tables IV and V. It should be noted that the application of the plate constants has really been effective in reducing the results to a homogeneous system. If the mean log E be formed for each star, and then the deviations of the individual values from these means, we obtain data for the calculation of any residual systematic differences between the plates. Excluding B.D. + 10°1771, which is peculiar, the results are

Plate	5210	5241	5244	5254	5250	5266	
Systematic Deviation	−0.013	+0.004	−0.002	+0.007	−0.005	+0.005	log E
	−0.032	+0.010	−0.005	+0.017	−0.012	+0.012	mag.

The average value of the probable error of a single determination of color from these measures, including residual errors in the plate constants, is $\pm 0^{M}066$.

TABLE IV

Colors from Exposure-Ratios, Region 6^h–7^h

Object	Vis. Mag.	Sp.	Log *E* (Blue-Yellow)						Color Excess (Unit, 1 Spectral Subdivision)					
			5209	5210	5241	5244	5254	Mean	5209	5210	5241	5244	5254	Mean
Nebulous stars														
N.G.C. 1514	9.0	B8	9.09	......	......	......	......	9.09	+1.2	......	......	......	......	+1.2
2175	7.40	Oe5	......	......	8.87	......	8.87	8.87	......	......	+1.0	......	+1.0	+1.0
2245	11.	A0	......	9.14	9.15	9.16	9.18	9.16	......	+1.3	+1.3	+1.4	+1.5	+1.4
2247	8.5	......	......	9.14	9.07	9.08	9.07	9.09	......	......	......	......	......	......
2282	9.7	B5	......	......	......	8.97	8.95	8.96	......	......	......	+0.7	+0.6	+0.6
I.C. 444	7.03	B9	......	......	9.15	9.09	9.11	9.12	......	......	+1.4	+1.1	+1.2	+1.2
446	10.5	B5	......	......	9.19	9.09	9.11	9.13	......	......	+2.1	+1.5	+1.6	+1.7
B.D.+10° 1771	8.0	K0	......	9.46	9.29	9.27	9.31	......	......	+0.1	−0.9	−1.0	−0.7	......
− 6 1415*	10.2	B1	......	......	......	9.21	9.24	9.22	......	......	......	+2.6	+2.8	+2.7
− 6 1417	9.1	B9	......	......	9.09	9.09	9.05	9.08	......	......	+1.1	+1.1	+0.8	+1.0
− 6 1418	9.1	B5	......	......	9.08	9.14	9.15	9.12	......	......	+1.4	+1.4	+1.8	+1.5
− 6 1431†	9.1	B5	......	......	8.99‡	9.15	9.06	9.07	......	......	+0.08	+1.8	+1.3	+1.3
−10 1848	7.01	B0	......	......	......	9.09	9.06	9.08	......	......	......	+2.0	+1.8	+1.9
−12 1771	8.5	B9	......	......	8.91	8.87	......	8.89	......	......	−0.1	−0.4	......	−0.2
													Mean	+1.28
Check stars														
Boss 1751	6.0	B8	6 B	8.91	8.90	8.94	8.83	8.90	......	0.0	−0.1	+0.2	−0.5	−0.1
1788	6.0	B8	and	and	8.89	8.91	8.87	8.89	......	......	−0.1	0.0	−0.2	−0.1
1944	2.9	B8	11 A	9	8.90	8.94	9.03	8.96	......	......	−0.1	+0.2	+0.8	+0.3
2008	0.2	F3	stars	other	......	9.15	9.17	9.16	......	......	......	0.0	+0.1	+0.1
2020	6.0	Mb	......	B stars	9.56	9.50	9.59	9.55	......	......	+0.1	−0.3	+0.2	0.0
2330	4.4	B3	......	......	8.89	8.88	8.83	8.87	......	......	+0.3	+0.2	−0.1	+0.1
													Mean	+0.05

* N.G.C. 2170. † N.G.C. 2182. ‡ Images bad.

Spectra of N.G.C. 1514 and 2245 by Sanford with focal plane slit spectrograph and 100-inch reflector; N.G.C. 2282 and I.C. 446, 6° objective prism and 10-inch refractor (uncertain); B.D. −6°1415 and −6°1418, 15° objective prism and 10-inch refractor; spectra of other nebulous stars are from the *Revised Draper Catalogue;* those of check stars are by Adams and from the *Revised Draper Catalogue.* In all cases the spectrum refers to the star—not to the surrounding nebulosity.

Magnitudes of N.G.C. 2245 and I.C. 446 are estimates; those of N.G.C. 2175, I.C. 444, and B.D. −10° 1848 are from *Harvard Annals,* **54;** those for the other nebulous stars are B.D. values corrected by *Harvard Annals,* **72.**

The very large color excess for B.D. −6°1415 seems to be real; but the result requires confirmation, for the images are very faint on both plates.

TABLE V

COLORS FROM EXPOSURE-RATIOS, REGION 16^h–18^h

Object	Vis. and Pv. Magnitude	Sp.	Log E			Color Excess		
			5250	5266	Mean	5250	5266	Mean
Nebulous stars								
ρ Ophiuchi Bt.	5.22	B5	9.05	9.00	9.03	+1.2	+0.9	+1.1
ρ Ophiuchi Ft.	5.92		9.06			+1.3		
σ Scorpii	3.08	B1	9.05	8.94	9.00	+1.6	+0.9*	+1.2
22 Scorpii	4.87	B5	9.88	8.89	9.88	+0.1	+0.2	+0.2
B.D. −10°4713	5.80	B5	9.03	9.11	9.07	+1.1	+1.6	+1.4
−19 4357	6.2	G5	9.33	9.31	9.32	−0.1	−0.2	−0.2
−19 4359 Bt.	7.1	B9	8.99	9.01	9.00	+0.4	+0.5	+0.4
−19 4359 Ft.	8.0	A0	9.05	8.99	9.02	+0.7	+0.3	+0.5
−19 4361 Bt.	7.0	B9	9.07	9.07	9.07	+0.9	+0.9	+0.9
−19 4361 Ft.	8.4	A0	9.06	9.07	9.06	+0.8	+0.8	+0.8
−19 4940	9.6	B3	......	8.99	8.99	......	+1.0	+1.0
−19 4943	9.0	B9	......	9.05	9.05	......	+0.8	+0.8
−19 4946 Bt.	10.9	B3	......	9.01	9.01	......	+1.1	+1.1
−19 4946 Ft.	10.9	B3	......	8.97	8.97	......	+0.9	+0.9
−19 4948	9.0	B1	......	8.95	8.95	......	+1.0	+1.0
−19 4953	7.5	B5	......	9.02	9.02	......	+1.0	+1.0
−19 4954	9.9	......	......	9.37	9.37	......	......	......
C.D. −23 12860	6.56	B8	9.09	9.07	9.08	+1.1	+1.3	+1.2
−23 12862	......	......	9.09	9.09	9.09	......	......	......
−23 13908	10.1	B3	9.09	8.97	9.03	+1.6*	+0.9*	+1.2
−23 13983	9.6	B5	......	9.14	9.14	......	+1.8†	+1.8
−23 13997	9.1	B5	8.95	8.96	8.96	+0.6	+0.6	+0.6
−23 13998	8.6	B3	8.93	8.97	8.95	+0.6	+0.9	+0.8
−23 13999	9.9	A2	9.00	8.96	8.98	+0.2	−0.1	0.0
−23 14002	9.9	A2	8.95	8.99	8.97	−0.1	+0.1	0.0
−23 14004	9.7	......	8.99	8.95	8.97	......	......	......
−23 14005	7.4	B3	8.97	8.98	8.98	+0.7	+0.7	+0.7
−23 14017	8.5	B5	8.97	9.04	9.00	+0.7	+1.1	+0.9
−24 12684	8.0	B5	9.18	9.13	9.16	+2.0	+1.7	+1.8
−24 13962	7.2	B1	8.97	8.97	8.97	+0.9	+1.1	+1.0
−24 13984	9.8	B1	......	9.08	9.08	......	+1.7	+1.7
							Mean	+0.88
Check stars								
Boss 3758	3.9	F3	9.17	9.09	9.13	−0.1	−0.6	−0.4
3810	5.8	K0	9.54	9.45	9.50	+0.8	+0.2	+0.5
4137	5.6	F9	9.24	9.28	9.26	−0.1	+0.1	0.0
4193	0.8	Ma	9.54	9.61	9.58	−0.1	+0.4	+0.2
4731	4.8	F3	9.09	9.03	9.06	−0.2	−0.6	−0.4
5003	5.1	B0	8.82	8.91	8.86	+0.1	+0.7	+0.4
5062	0.6	A4	9.03	......	9.03	−0.2	......	−0.2
							Mean	+0.01

* Images bad.

† Images at edge of plate.

NOTES TO TABLE V

For those stars included in the polar comparisons given in Tables VI and VII the magnitudes are photo-visual. Magnitudes for the remaining nebulous stars are visual, from *Harvard Annals*, **50** or **54**, and the C.D. corrected by *Harvard Annals*, **72**.

ρ Ophiuchi: Sp. from *Harvard Annals*, Frost and others, is probably of brighter component only.

σ Scorpii: Sp. from *Harvard Annals*, Campbell and others. A spectroscopic binary with stationary H and K lines. *Harvard Annals*, **76**, gives a color excess of $+0^{M}33$ mag.

22 Scorpii, and B.D. $-10°4713$: Sp. from *Harvard Annals*, checked by MW slit spectrogram.

B.D. $-19°4357$: Slit spectrogram, 60-inch. Abs. mag. by Adams, one plate, -0.3.

B.D. $-19°4359$, $-19°4361$: Sp. from *OP* 10-inch refractor, and *FP* slit, 60-inch reflector.

B.D. $-19°4940$: Sp. from *OP* 10-inch refractor. Lines very diffuse.

B.D. $-19°4943$: Sp. from *OP* 10-inch refractor. Lines very wide, especially Hδ.

B.D. $-19°4946$: Sp. from *FP* slit, 60-inch reflector.

B.D. $-19°4948$: Sp. from *OP* 10-inch refractor.

B.D. $-19°4953$: Sp. from *FP* slit, 60-inch reflector.

C.D. $-23°12860$: $3'$ *p* ρ Ophiuchi. Sp. from *OP* 10-inch refractor.

C.D. $-23°12862$: $2'$ N ρ Ophiuchi.

Spectra of remaining nebulous stars from *OP* 10-inch refractor.

C.D. $-23°13908$: Sp. very faint.

C.D. $-23°13983$: Lines widely double.

C.D. $-23°14005$ and $-24°13962$: Sp. checked by *FP* slit, 60-inch reflector.

Spectra of check stars are by Adams or from *Harvard Annals*.

The significant quantities are those in the two columns headed "Color Excess." These are expressed in spectral subdivisions and were obtained by subtracting the spectral type from the color-class corresponding to log E. Their values in magnitudes on the color system of the reflector may be found by multiplying by 0.4. Thus for N.G.C. 1514, Plate 5209, log $E=9.09$; the color-class is *f*0 and the spectrum B8, whence the color excess is $+1.2$, or in magnitudes, $+0.48$.

The values of the color excess for the check stars are merely residuals, inasmuch as the color of these stars is assumed to be normal. In contrast with these, the large and persistently positive values of the color excess for the nebulous stars show that their colors are well in advance of their spectral types. The phenomenon seems to be somewhat more pronounced for the group in 6^{h}–7^{h} than for that in the later hours of right ascension, the respective means being $+1.28$ and 0.88. The corresponding values in magnitudes are $+0.51$ and $+0.35$.

The star B.D. $+10°1771$, $8^{M}_{.}0$, is not nebulous, but follows the nebulous star N.G.C. 2245 closely and lies in the dark lane connecting this object with N.G.C. 2247. The spectrum, Ko, is from the *Revised Draper Catalogue*. The values of log E are subject to a systematic correction because of the size of the images, the exposures and aperture having been adjusted to N.G.C. 2245 and I.C. 446, which are both in the same field and much fainter; but their relative values should be reliable. The accordant results from Plates 5241, 5244, 5254 can be reconciled with the spectrum only by supposing the star to be of low luminosity, $M=+6$ or fainter. The value from Plate 5210, on the other hand, indicates high luminosity. The images on this plate are excellent. The possibility of an error in the record which would explain the discordance seems to be excluded by the results for N.G.C 2245 and 2247 (observed simultaneously with B.D. $+10°1771$), which are in agreement with those from the other plates. The behavior of the star suggests variability.

These results have been tested by a direct determination of the color-indices of two groups of nebulous stars in $18^{h}\,5^{m}_{.}5, -24°$ and $18^{h}12^{m}, -19°$, respectively, with the aid of polar comparisons made with the 10-inch Cooke triplet. One pair of plates for the determination of photographic and photo-visual magnitudes was made for each region; and, as a check on the zero-points of the magnitude scales, several stars near the nebulous objects, but not involved in nebulosity, were also included in the comparison. The results are given in Tables VI and VII.

The photographic and photo-visual magnitudes are on the Mount Wilson system,[1] reduced to the color system of the 10-inch refractor by the formula

$$Pg_{10}=Pg_{60}-0.19\,C_{60},$$

in which Pg_{60} represents the photographic magnitude of a polar standard on the color system of the reflector and C_{60} its corresponding color-index. The photo-visual magnitudes require no reduction.

[1] *Mt. Wilson Contr.*, No. 97; *Astrophysical Journal*, **41**, 206, 1915.

The spectra are from an objective prism plate with the same instrument. The relation between color-index and spectrum for the check stars is shown in Figure 1, from which it appears that the zero-points of the magnitude scales are well determined. For Region II there is a slight systematic effect, but for Region I

TABLE VI

COLORS BY POLAR COMPARISONS. REGION I, R.A. 18^h5^m5 DEC. $-24°$

Object	Magnitude		Color-Index	Color-Class	Sp.	Color Excess	
	Pg.	Pv.				CC−Sp.	Table V
Nebulous stars							
B.D. −22°4510*	8.04	7.25	+0.79	*g*5	A3	+2.2	....
C.D. −23 13836	8.34	8.74	−0.40	*o*8	B0	−0.2	....
−23 13908	10.23	10.10	+0.13	*a*4	B3	+1.1	+1.2
−23 13954	9.64	9.55	+0.09	*a*3	B3	+1.0	....
−23 13983	9.78	9.64	+0.14	*a*4	B5	+0.9	+1.8†
−23 13997	9.08	9.10	−0.02	*b*9	B5	+0.4	+0.6
−23 13998	8.45	8.56	−0.11	*b*7	B3	+0.4	+0.8
−23 13999	10.02	9.87	+0.15	*a*5	A2	+0.3	0.0
−23 14002	9.95	9.82	+0.13	*a*4	A2	+0.2	0.0
−23 14004	9.82	9.73	+0.09	*a*3‡	....	....	....
−23 14005	7.64	7.38	+0.26	*a*8	B5	+1.3	+0.7
−23 14017	8.47	8.46	+0.01	*a*0	B5	+0.5	+0.9
−24 13962	7.38	7.20	+0.18	*a*6	B1	+1.5	+1.0
−24 13984	9.93	9.78	+0.15	*a*5	B1	+1.4	+1.7
Check stars							
C.D. −23°13909	9.45	8.37	+1.08	*k*4	K2	+0.2	
−23 13991	7.62	7.60	+0.02	*a*1	A2	−0.1	
−23 13881	7.[illegible]7	8.01	−0.34	*o*9	B2	−0.3	
−24 13893	9.03	8.96	+0.07	*a*2	A2	0.0	
−24 13880	7.54	7.65	−0.11	*b*7	B5	+0.2	
−23 14016	8.80	8.42	+0.38	*f*1	F2	−0.1	
−23 14012	9.43	9.37	+0.06	*a*2	B9	+0.3	
−23 14001	9.08	8.19	+0.89	*g*8	K0	−0.2	
Systematic difference for check stars						0.00	

* The bright central star in the northern mass of the Trifid nebula.

† Images at edge of plate. ‡ Color-class from exposure-ratio *a*2.

spectrum A0 corresponds to zero color-index very accurately. For K0, $C_{10}=0.96$. For the reflector the corresponding value is $C_{60}=1.20$, whence the coefficient of the color equation is 0.20, in agreement with the value given above.

Assuming the increase in color to be linear, we find from the color-indices by the formula, color-class $=C_{10}/0.32$, the values given in the fifth column of Tables VI and VII. The differences,

color-class *minus* spectrum, expressed in spectral subdivisions, are the values of the color excess. The mean differences for the check stars are small, as was to have been expected. Their values are 0.00 and +0.14, respectively, or in magnitudes, 0.00 and +0.03. The nebulous stars, on the other hand, with the exception of C.D. $-23°13836$, 13999, and 14002 (Table VI), show large values of the color excess, in agreement with the results obtained from the exposure-ratios, which are inserted in the last column of Tables VI and VII. Correcting the values of

TABLE VII

COLORS BY POLAR COMPARISONS, REGION II, R.A. 18^h12^m DEC. $-19°$

Object	Magnitude		Color-Index	Color-Class	Sp.	Color Excess	
	Pg.	Pv.				CC−Sp.	Table V
Nebulous stars							
B.D. −19°4940......	9.55	9.50	+0.05	*a*2	B3	+0.9	+1.0
−19 4943......	9.09	9.00	+0.09	*a*3	B9	+0.4	+0.8
−19 4946 Bt....	10.88	10.76	+0.12	*a*4	B5	+0.9	+1.1
−19 4946 Ft....	10.91	10.90	+0.01	*a*0	B3	+0.7	+0.9
−19 4948......	8.75	8.80	−0.05	*b*9	B1	+0.8	+1.0
−19 4953......	7.56	7.49	+0.07	*a*2	B5	+0.7	+1.0
−19 4954......	10.78	9.80	+0.98	*k*1*	......	......	......
Check stars							
B.D. −19°4970......	7.91	7.75	+0.16	*a*5	A3	+0.2	
−19 4965......	9.07	8.44	+0.63	*g*0	F8	+0.2	
−19 4969......	9.21	9.00	+0.21	*a*7	A2	+0.5	
−19 4952......	7.98	7.62	+0.36	*f*1	F2	−0.1	
−20 5065......	9.24	9.30	−0.06	*b*8	B5	+0.3	
−19 4935......	9.45	9.22	+0.23	*a*7	A5	+0.2	
−19 4929......	8.56	8.09	+0.47	*f*5	F8	−0.3	
	Systematic difference for check stars					+0.14	

* Color-class from exposure-ratio *g*6.

the color excess derived in Table VII for the small outstanding systematic difference shown by the check stars, which is probably to be attributed to zero-point errors in the magnitudes, we find means for the stars observed by both methods as follows:

	From C.-I.	From log E	No. of Stars
Table VI..........	0.80	0.87	10
Table VII.........	0.59	0.97	6

Summarizing the results we find:

	Nebulous Objects Observed	Spectra Lacking	Positive Effect	Exceptions	Doubtful
Table IV	13	1	11	B.D. -12° 1771	....
Table V	31	4	23	B.D. -19 4357	22 Scorpii
				C.D. -23 13999	....
				C.D. -23 14002	....
Additional, Table VI	3	....	2	C.D. -23 13836	....
Totals	47	5	36	5	1

Of the forty-two objects for which complete data are available, thirty-six show a positive effect whose reality can scarcely be doubted. A revision of the spectral classification may modify

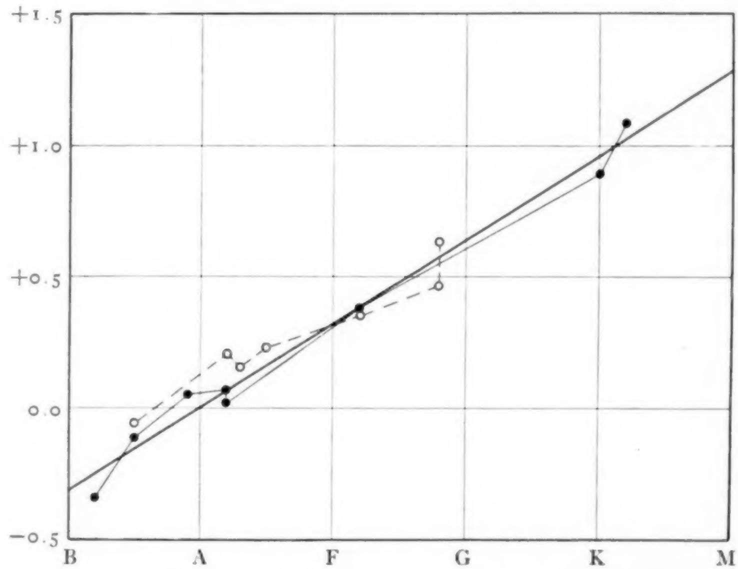


FIG. 1.—Spectral type and color-index for the 10-inch Cooke refractor. Full line, Table VI; broken line, Table VII.

the number slightly; but the close accordance of the spectra for the check stars in Tables VI and VII and the corresponding color-indices (see Fig. 1) lead us to hope that no serious uncertainty arises from this source. As for the exceptional cases, it has already

been remarked that, in some cases at least, a close association with nebulosity has not certainly been established. In any event it seems clear that a large majority of these objects are approximately one color-class in advance of their spectral types.

It would be premature at this stage of the investigation to express an opinion as to the cause of the phenomenon. An explanation that will occur to everyone, perhaps the most plausible at hand, is that of molecular scattering produced by the nebulosity surrounding the star. But there is also the chance of some form of fluorescent excitation in the nebulous material closely adjacent to the stars; and one should likewise bear in mind the possibility that peculiar internal conditions may occur in stars intimately involved in nebulosity, producing something similar to the luminosity effect that is now well established for the G and K stars at large. Narrowly considered, this last suggestion does not seem immediately applicable. Almost without exception the nebulous stars have B or early A spectra; and the luminosity effect for these types, if present at all, is certainly much less than the color excess observed in the nebulous stars. But if we think of the luminosity effect upon color as being primarily atmospheric in origin, and if at the same time we look upon the luminous material surrounding the nebulous stars as an integral part of its atmosphere, chaotic and unorganized through a neutralization of gravitational attraction by repulsive forces, the connection may not be so remote. This, however, leads us back to the suggestion of molecular scattering as the primary phenomenon.

Finally, a scattering by particles of more than molecular size should not be forgotten. In this connection it is at least suggestive that the nebulosity surrounding N.G.C. 2245, 2247, 2282, and I.C. 446,[1] as well as that covering the region of ρ Ophiuchi, shows a continuous spectrum. Moreover, all through that part of the sky including the group of nebulous stars in 16^h–18^h of right ascension we find extended areas of low star density, dark lanes, and vacant patches that seem to indicate an obstruction of starlight by masses of non-luminous obscuring material. These regions are too well known through the observations of Barnard and others to

[1] The nebulosity about N.G.C. 1514, on the other hand, is gaseous.

need comment. It may be pertinent to remark, however, that ρ Ophiuchi and the two seventh-magnitude stars near it, C.D. $-23°12860$ and 12862, are the only objects brighter than the tenth magnitude in an area of more than three square degrees.[1] The average number to this limit per square degree in the regions immediately adjoining is from 10 to 15.

These circumstances may have some bearing upon the exceptional cases of nebulous objects which show no appreciable excess of color. If the particles of the nebulous mass be above a certain size, the characteristic phenomenon of scattering does not occur, and we have a reduction in the intensity of the transmitted light which affects all wave-lengths equally. It is instructive in this connection to attempt to correlate the excess of color with the density of the surrounding nebulosity. Subdividing the stars into three groups, we find the following mean values of the color excess:

Nebulosity	Color Excess	No. of Stars	No. of Exceptions
Moderate........	+1.0	27	4
Dense...........	+1.5	6	0
Very Dense......	+0.9	9	2
All..............	+1.04	42	6

There is no certain evidence of correlation; but it is perhaps significant that of the nine stars involved in very dense nebulosity, two are exceptional or doubtful, and four others have values of the color excess below the average for the entire group of forty-two stars.

Mount Wilson Observatory
May 14, 1920

Note.—Photographs taken with the 60-inch and 100-inch reflectors after the foregoing paper was written show C.D. $-23°13999$, 14002, and 14004 (Table VI) apparently superimposed upon a background of smooth nebulosity, which does not condense about the stars. This is also the case with B.D. $-19°4954$ (Tables V and VII). The nebulous nature of these objects is therefore doubtful.

[1] See the C.D. charts.

The condensation about B.D. $-19^\circ 4357$ (Table V) is very feeble, especially as compared with that of its neighbors B.D. $-19^\circ 4359$ and 4361.

22 Scorpii (Table V) itself is apparently free from nebulosity, although two intricate streamers bend around it to form what Professor Barnard calls "the eye nebula," of which 22 Scorpii is the pupil. The symmetrical arrangement of the streamers suggests a repulsive force emanating from the star. Professor Barnard has published an excellent photograph of the region, taken with the Bruce 10-inch camera, in *Astrophysical Journal*, **31**, 10, 1910.

A recent photograph of C.D. $-23^\circ 13836$ shows that the star is not nebulous.

B.D. $-12^\circ 1771$ alone therefore remains as an exception of a real and very striking nature.

Recent exposure-ratio photographs confirm the very large color excess found by polar comparisons for B.D. $-22^\circ 4510$ (Table VI), and place it in color-class g4. Additional spectrograms leave no doubt that the spectrum is really A3. The lines are normal in distribution and number, and there is no trace of bright-line radiation in the red. The very pronounced color, which visually is in striking contrast to that of the companion star B.D. $-23^\circ 338$ in the principal mass of the Trifid nebula, must therefore be attributed to peculiarities in the continuous spectrum.

June 28, 1920.

ON THE DISTRIBUTION OF THE STARS IN SPACE ESPECIALLY IN THE HIGH GALACTIC LATITUDES[1]

BY J. C. KAPTEYN[2] AND P. J. VAN RHIJN

ABSTRACT

Distribution of stars in space.—While the results reported in this paper are provisional, in that some recently available parallaxes are not included and the investigation of the lower galactic latitudes has not been completed, it is unlikely that more complete data will materially modify them. (1) *In galactic latitudes* $\pm 40°$ *to* $\pm 90°$. The *average parallax for all stars of a given magnitude* (m) *and proper motion* (μ) is found to be represented satisfactorily by the formula $\log \pi = -0.691 - 0.0682\, m + 0.645 \log \mu$. By combining this result with the law of dispersion of parallax for stars of given m and μ, which had already been determined, the two fundamental laws which determine the arrangement of stars in space were found. The first of these is the *luminosity-curve* or frequency of the several absolute magnitudes per unit volume, which, at least near the sun, is found to correspond accurately, from $-10^{M}6$ to $+7^{M}4$, to a symmetrical probability-curve with the equation: $\log \phi(M) = -2.394 + 0.1858M - 0.03450M^2$ (see Table IV and Fig. 1). Assuming that the luminosity-curve is the same for all distances from the sun, the median absolute magnitude of all stars is 2.7 (which is about 2.9 magnitudes fainter than the sun), with a probable error of $\pm 1^{M}69$. The second law is the *law of stellar densities* as a function of parallax. For distances below 1000 parsecs the densities can be determined directly (see Table V). For greater distances the following formula was derived from the luminosity-curve and the distribution-curve of apparent magnitudes, assuming no extinction of light in space: $\log \Delta(\rho) = -3.425 + 3.526 \log \rho - 0.943 (\log \rho)^2$ (see Table VI). (2) *In galactic latitudes* $0°$, $30°$, $60°$, *and* $90°$. A less refined investigation of all stars in these latitudes has led to formulae for the densities as a function of parallax which are similar to that given above (see Tables VI and VII). These results enable us to draw a *section of the galactic system* at right angles to the galactic plane, with the sun at the center (Fig. 2), which shows that in the direction of the galactic poles about 1500 parsecs may be taken as practically the limit of the system, while in a direction in the plane of the Milky Way the same small density is eight times more distant. The authors point out that, since symmetry around the galactic poles is assumed, this work is merely a second approximation to the complete solution of the problem.

1. The investigations contained in *G.P.*[3] 27, 29, and 30 were carried out for the purpose of making possible an elaborate treatment of the arrangement of the stars in space. At least two more

[1] *Contributions from the Mount Wilson Observatory*, No. 188.

[2] Research Associate of the Mount Wilson Observatory.

[3] By *G.P.* we will denote the *Publications of the Astronomical Laboratory at Groningen.*

publications will be necessary to complete this investigation. Now that, after so many years of preparation, our data seem at last to be sufficient for the purpose, we have been unable to restrain our curiosity and have resolved to carry through completely a small part of the work, even though, by so doing, the rules for strict economy of labor cannot be altogether adhered to.

The present paper is the outcome of this more or less provisional work. In the main it relates to the stars as a whole between galactic latitudes $\pm 40°$ and $\pm 90°$. It is provisional in that the extremely valuable parallaxes now placed at our disposal by Mitchell have not yet been used, and moreover because the thorough discussion which will have to be made of the measured parallaxes in all galactic latitudes is not yet completed. We expect little to be changed, however, in the definitive treatment of the subject, which will form a part of the more comprehensive work.

A somewhat less refined investigation of all the stars (irrespective of spectrum) in galactic latitudes 0°, 30°, 60°, and 90° has also been added.

2. In *G.P.* 30 were found the numbers of stars for each magnitude down to $m = 12.0$ and for every value of the proper motion μ. These numbers, corrected for observational errors, are contained in Table 19, which also gives for magnitudes 13 and 14 the numbers for proper motions exceeding $0''.200$.

We begin by deriving the average parallax for each of these classes of stars, i.e., for stars having given values of m and μ. Following the method indicated in *G.P.* 8, we tentatively start from the formula arrived at in that publication,

$$\log \pi_{m,\mu} = A + Bm + C \log \mu. \qquad (1)$$

It turns out that this formula represents all our data satisfactorily. It is true that some of the divergences are larger than seems desirable, but there is reason to believe that the defect is not in the formula. At all events, the question whether a still more satisfactory formula can be assigned can be settled only by the later definitive treatment of the data for the whole of the sky.

The measured parallaxes available for our investigation embrace all the published results that seem to deserve confidence, excluding

those whose probable errors exceed $0''.025$. In addition we are indebted to Professor Schlesinger for a magnificent list of unpublished determinations. We cordially thank him for this generous act of courtesy. Professor Mitchell, too, as already mentioned, has communicated his invaluable results. These will, of course, play a prominent part in our more elaborate work, but they arrived too late for the present note.

For those cases in which several determinations are available for the same star, weights were adopted, not exactly in accordance with the probable errors as given by the authors, but modified somewhat by the supposition that small systematic errors still attach to all the results.

For proper motions exceeding $1''.00$, the stars in all galactic latitudes were used. For smaller motions we confined ourselves for the present to galactic latitudes higher than 40°.

In order to find the change of π with μ (the constant C in (1)) we divided the stars with measured parallaxes into two groups according to brightness—magnitudes 3.5 to 6.5, and those that are fainter. Both these groups were subdivided according to proper motion. We thus obtained the normal values in Tables I and II, which, by means of small corrections derived from *G.P.* 8, were all reduced to magnitudes 5.0 and 8.0.

The data obtained from direct parallax determinations given in these tables were supplemented by results derived from the parallactic motions in Table 25 of *G.P.* 29. The latter parallaxes were obtained by adopting 19.5 km for the sun's velocity, and are entered in the last lines of Tables I and II.

The constants in formula (1) were then determined in such a way that the values from the parallactic motions are almost exactly represented, while at the same time the measured parallaxes are represented as well as possible. The results thus found are

$$\log \pi_{5.0} = -1.040 + 0.630 \log \mu \tag{2}$$

$$\log \pi_{8.0} = -1.163 + 0.659 \log \mu . \tag{3}$$

The parallaxes computed with these formulae have been designated by π_1 in Tables I and II. Although the residuals $O-\pi_1$

may not seem to be all that can be desired, we believe our tables prove that, at least as far as the change with proper motion is concerned, formula (1) cannot be materially in error. As we have already remarked, however, it seems best to reserve a closer dis-

TABLE I

MAGNITUDE 3.5 TO 6.5

m	True μ	π	No.	π_1	π_2	$O-\pi_1$	$O-\pi_2$
5.0.....	0."048	+0."018	31	0."0135	0."013	+0."0045	+0."005
5.0.....	.123	+ .034	23	.0245	.024	+ .0095	+ .010
5.0.....	.251	+ .043	27	.038	.038	+ .005	+ .005
5.0.....	.436	+ .047	15	.054	.054	− .007	− .007
5.0.....	.670	+ .072	18	.071	.072	+ .001	.000
5.0.....	1.260	+ .134	23	.105	.108	+ .029	+ .026
5.0.....	3.77	+ .174	9	.211	.218	− .037	− .044
5.0 (mean) *G.P.* 29..		+0.0204	....	0.0200	0.0192	+0.0004	+0.0012

TABLE II

MAGNITUDE 6.5 AND FAINTER

m	True μ	π	No.	π_1	π_2	$O-\pi_1$	$O-\pi_2$
8.0.....	0."202	+0."021	29	+0."024	0."021	−0."003	0."000
8.0.....	.486	+ .032	22	+ .043	.036	− .011	− .004
8.0.....	.714	+ .040	31	+ .055	.047	− .015	− .007
8.0.....	1.30	+ .085	28	+ .082	.069	+ .003	+ .016
8.0.....	4.05	+ .214	13	+ .173	.143	+ .041	+ .071
8.0 (mean) *G.P.* 29..		+0.0079	....	0.0079	0.0072	0.0000	+0.0007

cussion of this point until later. As the combined result of (2) and (3) we adopt

$$C=+0.645. \tag{4}$$

3. We derive the constants A and B exclusively from the mean parallaxes found from parallactic motions. Writing (1), in which C has the value (4), in the form

$$\pi=10^{A+Bm}\times 10^{0.645 \log \mu}, \tag{5}$$

we find for the mean parallax of all the stars of magnitude m,

$$\pi = 10^{A+Bm} \times \overline{10^{0.645 \log \mu}}, \tag{6}$$

where the dashes indicate average values.

Since the total number of stars for every value of m and μ is given in Table 19 of *G.P.* 30, we can at once find the average value occurring in the second member. The values of $\bar{\pi}$ are obtained from Table 25 (in which the sun's velocity = 19.5 km). Introducing logarithms, we find the equations of condition in Table III.

TABLE III

EQUATIONS OF CONDITION

Magnitude	Equation	$\log \pi_{\text{comp.}}$	$O-C$ $\log \pi_{29} - \log \pi_{\text{comp.}}$
4	$A+4B=-1.033$	-0.975	-0.058
5	$A+5B=-1.019$	-1.046	$+.027$
6	$A+6B=-1.121$	-1.118	$-.003$
7	$A+7B=-1.156$	-1.189	$+.033$
8	$A+8B=-1.215$	-1.260	$+.045$
9	$A+9B=-1.326$	-1.332	$+.006$
10	$A+10B=-1.429$	-1.403	$-.026$
11	$A+11B=-1.499$	-1.474	-0.025

Solved by least squares, these equations yield

$$A = -0.690 \qquad B = -0.0713, \tag{7}$$

so that finally

$$\log \pi_{m,\mu} = -0.690 - 0.0713\, m + 0.645 \log \mu, \tag{8}$$

with which the third column in Table III was computed. Instead of this formula, however, the one actually used in what follows is

$$\log \pi_{m,\mu} = -0.691 - 0.0682\, m + 0.645 \log \mu. \tag{9}$$

Since the two equations represent the observations almost equally well, we have not repeated the computations.

The coefficient of m corresponds to $\log \epsilon$ in *G.P.* 8, formula (3). Hence from (9), $\epsilon = 0.855$, whereas in *G.P.* 8 the value $\epsilon = 0.905$ was adopted. In the meantime this value of ϵ has already been corrected in *G.P.* 11 (p. 20) to $\epsilon = 0.87$, in good agreement with the present value.

The parallaxes obtained by means of (9) are given in Tables I and II under the heading π_2. The residuals for this definitive solution are given under the heading $O - \pi_2$. Only in the case of the largest proper motions are they somewhat excessive. That this is not due to a defect in our formula is proved by the fact that the sign in the case of magnitude 8.0 is the opposite of that for magnitude 5.0. For the present purpose we do not consider these residuals for the largest proper motions to be of prime importance, since they are extremely rare.[1] Had this not been the case we might have lessened these extreme values somewhat by adopting a slightly larger value of C.

4. Formula (9) contains the solution of the problem presented in section 2, viz., to find the average parallax for each of the classes of stars of given m and μ contained in Table 19 of *G.P.* 30. The stars of apparent magnitudes 13 and 14 in this table, all having large proper motions, must be of extremely low absolute magnitude; they are accordingly very valuable for extending the luminosity-curve at its fainter extremity. That the frequency of the smaller proper motions is not known for these stars is of small importance. Their influence on the computations actually used for what follows is so small that we can quite safely introduce extrapolated values.

Although by (9) we can find the *average* parallax of the stars of any m and μ, it is of course evident that the parallax of any single star of the given m and μ will in general differ from the mean parallax. But for the problem of the general arrangement of the stars in space it is not necessary to know the distance of each individual star. It suffices to know how many stars have such and such a distance. This knowledge may be obtained if, in addition to the average parallaxes already found, we can find the dispersion law of the parallaxes, that is, the law which, for the stars of given

[1] In further computations the best plan will probably be to use for these stars the individual parallaxes as found by direct determination.

m and μ, specifies, for any value of a, the frequency of a parallax a times the average parallax.

Having found the number of stars of given magnitude m and a given parallax, we shall know of course for these same stars the absolute magnitude M.

It has been shown in *G.P.* 11 (pp. 17–20) that widely differing assumptions as to the dispersion law lead to results that differ but little. In this preliminary solution, therefore, we have not deemed it necessary to derive this law anew, but have adopted the one found and tabulated in *G.P.* 8 (solution D).

Since it thus becomes possible to find, for any given distance, the number of stars of each absolute magnitude, we can evidently derive the two fundamental laws which determine the arrangement of the stars in space, viz., the luminosity-curve and the law of stellar densities. A convenient manner of conducting the computations is explained at length in *G.P.* 11 and need not be repeated here. It should be remembered, however, that it introduces the two assumptions: (1) there is no appreciable extinction of light in space; (2) the frequency of the several absolute magnitudes (luminosity-curve) does not change with the distance from the sun. These assumptions can be avoided, at least to a considerable extent. They will be discussed in the definitive solution.

Different units of distance have been used by different astronomers. That most widely used at present is the parsec. For the sake of uniformity we have resolved henceforth not only to use this unit but also to use the name, which is very convenient (though very ugly). To conform with this new unit, the value of the absolute magnitude, which must still be defined to be the magnitude of a star as it would appear at the unit of distance, must also be changed. Its numerical value will now be five less than it was according to former publications, and may be found by the formula

$$M = m + 5 \log \pi = m - 5 \log \rho. \tag{A}$$

The results obtained in the present case are in Tables IV and V. The table giving the densities will presently be extended to greater distances through the consideration of other data.

Table V will presently be extended to greater distances by the consideration of other data.

TABLE IV

LOG NUMBER OF STARS PER CUBIC PARSEC NEAR THE SUN

(Determining Luminosity-Curve)

M	$\log \phi(M)$	Computed	O−C
−11.64	1.2 −10	0.8 −10	+0.4
−10.64	1.6	1.7	− .1
− 9.64	2.55	2.61	− .06
− 8.64	3.373	3.425	− .052
− 7.64	4.148	4.173	− .025
− 6.64	4.843	4.851	− .008
− 5.64	5.474	5.461	+ .013
− 4.64	6.020	6.001	+ .019
− 3.64	6.493	6.473	+ .020
− 2.64	6.894	6.875	+ .019
− 1.64	7.215	7.209	+ .006
− 0.64	7.472	7.473	− .001
+ 0.36	7.662	7.669	− .007
+ 1.36	7.776	7.795	− .019
+ 2.36	7.836	7.853	− .017
+ 3.36	7.819	7.841	− .022
+ 4.36	7.737	7.761	− .024
+ 5.36	7.627	7.611	+ .016
+ 6.36	7.364	7.393	− .029
+ 7.36	7.200	7.105	+ .095
+ 8.36	7.00	6.75	+ .25
+ 9.36	6.4	6.3	+0.1

TABLE V

DENSITY (=1 NEAR SUN)

π	Density	π	Density
0.″00118	0.089	0.″0296	0.918
.00187	.179	.0469	1.000
.00296	.298	.0743	1.000
.00469	.451	.118	1.000
.00743	.600	.187	1.000
.0118	.760	0.296	1.000
0.0187	0.864	....	....

The result for the luminosity-curve may be considered to be the final result of this paper. Its general high accuracy appears in the agreement of the large overlapping portions of the independent curves found for different distances from the sun. For brevity's sake we omit the numbers which show this, but even without

them we gain an insight into the accuracy through the astonishingly close approach of the values in Table IV to the simple analytical curve

$$\log \phi(M) = -2.394 + 0.1858\,M - 0.03450\,M^2 . \tag{10}$$

The last column of the table shows the residuals.

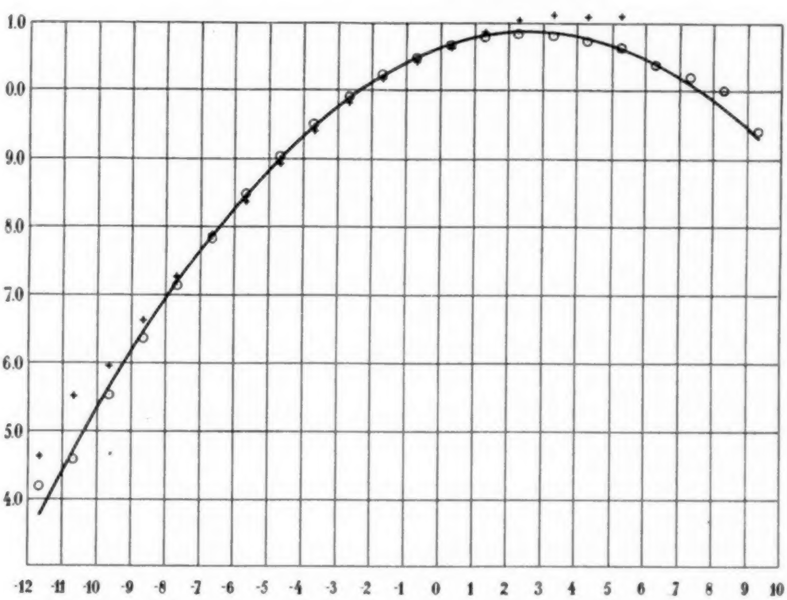


FIG. 1.—Luminosity-curve, all spectral types together. Abscissae are absolute magnitudes (unit of distance, 1 parsec). Ordinates are logarithms of numbers of stars per 1000 cubic parsecs near the sun.

A still better appreciation of the close approach of theory and observation may be obtained by inspecting Figure 1, which shows the curve (10) and the observed values, represented by small circles. With absolute magnitudes as abscissae, this curve gives the logarithm of the number of stars per 1000 cubic parsecs, instead of per cubic parsec, as does formula (10).

We may write (10) in the form

$$\phi(M) = A\frac{h}{\sqrt{\pi}}e^{-h^2(M-M_0)^2} \tag{11}$$

where

$$\left.\begin{array}{c} A=0.0451,\ M_0=2.693,\ h=0.2818 \\ r=\dfrac{0.4769}{h}=1.692 \end{array}\right\}. \qquad (12)$$

It thus appears that the luminosity-curve (or rather the absolute-magnitude curve) is represented very closely by an error-curve spread round the median value $M=2.7$, with a probable error of ± 1.69. If we assume that the agreement of the two curves holds for absolute magnitudes beyond those afforded by our data, the total number of stars per cubic parsec in the vicinity of the sun from the very brightest, absolutely, to the faintest is 0.0451.

The crosses in Figure 1 represent the results found in *G.P.* 11 from wholly different and very scanty material.[1] In *G.P.* 11 only the data for stars brighter than 6.0 or 6.5 may be considered as fairly reliable, whereas in the present paper most of the data are certainly trustworthy down to magnitude 11 (inclusive), and even down to magnitude 12 (inclusive) for the all-important proper motions of considerable amount, and begin to fail altogether only beyond magnitude 14.

Although the present curve is therefore infinitely preferable to the old one, it is still eminently satisfactory to see how closely the old curve agrees with the present result. The extremities were of necessity unreliable; the data on which it was based were defective not only in the proper motions and magnitudes but even more so in the measured parallaxes.

The most gratifying advance is the determination of the maximum of the curve. Owing to the inclusion of material for stars as faint as twelfth, thirteenth, and fourteenth magnitudes, we are at last well beyond the maximum, which can now be located with considerable precision. In many respects this will prove a very important gain, indeed, since it allows us for the first time to express with some certainty the relation between the average absolute magnitude of the stars and that of the sun. The absolute magni-

[1] Reduced from the Potsdam scale to the Harvard scale by subtracting 0.16 from the magnitudes.

tude of the sun[1] appears to be near -0.2, which is 2.9 magnitudes *brighter* than the average magnitude of all the stars.

If in Table IV we disregard the first two and the last two values, which are of necessity very uncertain, there still remain values ranging over no less than eighteen magnitudes, that is, over luminosities varying from one to sixteen millions, all of which lie with extreme precision on the same Gaussian curve. We doubt whether any other case is known in the whole domain of science of so close an agreement over so large a range—a range of more than ten times the probable error. It is difficult to avoid the conclusion that we have here to do with a law of nature, a law which plays a dominant part in the most diverse natural phenomena. This in itself would lead us to assume that the curve (11) will still hold for the remainder of the descending branch of the luminosity-curve. There is at least one other fact which tends to confirm our belief that this will be the case. For the B0, B1, B2, and B3 stars practically the whole curve is covered by the observations,[2] and no assured deviation from an error curve has been found. It is true that in this case the number of observations is very small. Be this as it may, it would be more satisfactory if the curve could still be prolonged somewhat by observation. We hope to consider elsewhere the possibility of doing so.

5. The densities in Table V do not extend to distances greater than a thousand parsecs and cannot claim any very great accuracy even for distances somewhat below this limit. It is evident that much would be gained by introducing a consideration of the known total number of stars of any given magnitude m.[3]

As before let $\phi(M)dM$ represent the luminosity-curve, i.e., the number of stars per cubic parsec near the sun having absolute magnitudes between M and $M+dM$. Further let $N_m dM$ represent the number of stars per 10,000 square degrees having apparent magnitudes m to $m+dm$. Finally let $\Delta(\rho)$ represent the total number of stars of any absolute magnitude per cubic parsec, taking

[1] *Astrophysical Journal*, **43**, 105, 1916.

[2] *Mt. Wilson Contr.*, No. 147, p. 72; for the B3 stars in particular see Table XXXIII.

[3] See *Proceedings Roy. Acad. Amsterdam*, March, 1908.

as unit the number in the neighborhood of the sun. It is easy to show, ρ being the distance, that

$$N_m = 0.9696\pi \int_0^\infty \rho^2 \Delta(\rho)\phi(m-5\log\rho)d\rho\ . \tag{13}$$

Schwarzschild[1] has shown that if

$$\phi(M) = e^{p+qM+rM^2} \tag{14}$$

and

$$N_m = e^{a+bm+cm^2} \tag{15}$$

we shall have

$$\Delta(\rho) = e^{h+k\log\rho+l(\log\rho)^2}\ . \tag{16}$$

Since (10) or (11) shows that $\phi(M)$ has indeed the form (14), and since it was long ago shown (see for example *G.P.* 27, p. 24) that with astonishing approximation N_m has also the form (15), we can find the density $\Delta(\rho)$ at once from (16). The relation between the constants is found to be

$$l = \frac{25cr}{r-c}, \tag{17}$$

and further, if we write

$$G = -l - 25r \tag{18}$$

$$k = 5q - 6.9078 - \frac{G(b-q)}{5r} \tag{19}$$

$$h = a - p - 2.5203 + \frac{\log G}{0.8686} - \frac{(k-5q+6.9078)^2}{4G}\ . \tag{20}$$

6. By means of these formulae we have computed the densities from the values of N_m in Table V of *G.P.* 27. Since the labor involved is small, we were not able to resist the temptation to treat in the same way the data for galactic latitudes 0°, 30°, 60°, and 90°, for this must give a first insight into the arrangement of the

[1] *Astronomische Nachrichten*, **185**, 81, 1910.

stars of the whole stellar system in space. In executing this plan we had of course to assume that the luminosity-curve is the same for all galactic latitudes, which seems little doubtful, although naturally we intend to investigate the matter in our definitive solution.

A little preliminary computation gave the following values of the constants by which the values of N_m in *G.P.* 27 are excellently represented:

Galactic Latitude	0°	30°	60°	90°	40° to 90°	Luminosity-Curve
a	−1.249	−1.296	−1.748	−2.079	−1.605	$p=-5.512$
b	+1.668	+1.593	+1.692	+1.783	+1.659	$q=+0.4279$
c	−0.0325	−0.0343	−0.0438	−0.0509	−0.0415	$r=-0.07944$

The constants of the luminosity-curve in accordance with (10) have also been added. Using these data we find by means of formulae (17) to (20):

Galactic Latitude	0°	30°	60°	90°	40° to 90°
h	−5.830	−5.193	−9.221	−14.319	−7.885
k	+5.705	+5.481	+9.320	+14.092	+8.120
l	−1.366	−1.508	−2.441	− 3.542	−2.171

which lead to the very convenient formulae

$$\left.\begin{array}{llll} \text{Gal. Lat. } 0°, & \log\Delta(\rho)=-2.532+2.478\log\rho & -0.593 & (\log\rho)^2 \\ 30, & \quad\quad\quad\;\; =-2.256+2.381 & -0.655 & \\ 60, & \quad\quad\quad\;\; =-4.005+4.048 & -1.060 & \\ 90, & \quad\quad\quad\;\; =-6.219+6.120 & -1.538 & \\ 40 \text{ to } 90, & \quad\quad\quad\;\; =-3.425+3.526 & -0.943 & \end{array}\right\} \quad (21)$$

The evident and well-recognized defect of these solutions is that the formula gives $\Delta(0)=0$, which is inadmissible. The values of $\Delta(\rho)$ furnished by (16) or (21) for the smaller distances cannot therefore be accepted. Fortunately our former solution gives good values for these very distances. The comparison of the results

obtained by the two methods, which can be made for galactic latitudes 40° to 90° and which is given below, proves that the

TABLE VI

Log $\Delta(\rho)$

Log ρ	ρ in Parsecs	Latitude				Latitude 40° to 90°		
		0°	30°	60°	90°	Formula (16)	Table V	Adopted
1.0	10.0	0.00	0.00	0.00	0.00	9.16	0.00	0.00
1.2	15.8	.00	.00	.00	.00	.45	.00	.00
1.4	25.1	.00	.00	.00	.00	.66	9.99	9.99
1.6	39.8	.00	.00	.00	.00	.80	.96	.96
1.8	63.1	.00	.00	.00	.00	.87	.92	.92
2.0	100	.00	9.89	9.85	9.87	.85	.84	.84
2.2	158	.00	.81	.77	.80	.77	.73	.75
2.4	251	.00	.68	.60	.61	.61	.59	.60
2.6	398	9.90	.51	.35	.30	.37	.39	.38
2.8	631	.76	.28	.02	8.86	.06	.14	.08
3.0	1000	.56	8.99	8.60	.30	8.67	....	8.67
3.2	1580	.32	.66	8.10	7.62	8.20	....	8.20
3.4	2510	.04	.27	7.50	6.81	7.66	....	7.66
3.6	3980	8.70	7.83	6.83	5.88	7.05	....	7.05
3.8	6310	.32	.33	6.07	4.83	6.36	....	6.36
4.0	10000	7.89	6.79	5.23	3.65	5.59	....	5.59
4.2	15800	7.42	6.19	4.30	2.35	4.75	....	4.75
4.4	25100	6.89	5.54	3.28	0.93	3.83	....	3.83
4.6	39800	6.32	4.84	2.19	9.39	2.84	....	2.84
4.8	63100	5.70	4.08	1.00	7.72	1.77	....	1.77
5.0	100000	5.03	3.27	9.74	5.93	0.63	....	0.63

TABLE VII

ρ

$\Delta(\rho)$	0°	30°	60°	90°
1.00	0	0	0	0
0.40	910	320	250	250
.16	1450	710	490	450
.063	3550	1320	800	660
.025	5750	2140	1200	910
.010	8910	3310	1700	1230
.0040	13200	4900	2340	1580
.0016	19100	7080	3090	2000
.00063	26900	10000	4070	2510
.00025	37200	13500	5130	3090
0.00010	50100	18200	6460	3720

present method yields good results as soon as the maximum is well passed.

For distances less than 100 parsecs[1] we have accordingly adopted the first solution for galactic latitudes 40° to 90.° For distances beyond this limit the two solutions give results which agree surprisingly well, and we have adopted the mean of the two. Beyond 630 parsecs the second solution is the only one available. For galactic latitudes 0°, 30°, 60°, and 90° the first solution has not been carried out. In close agreement with the result just found for the high galactic latitudes, we have assumed the density to be constant up to the maximum of the second solution, while, from that value on, the second solution was adopted. Table VI shows the values thus obtained, from which by interpolation we derive Table VII.

Figure 2 was constructed with the aid of Table VII. If we imagine this figure completed by the addition of its reflected image on the other side of the line AB, it will represent a complete section through the galactic system at right angles to the galactic plane. The lines shown are lines of constant density. The sun lies at the center S. The numbers along the bottom line show the distances expressed in parsecs. The density is

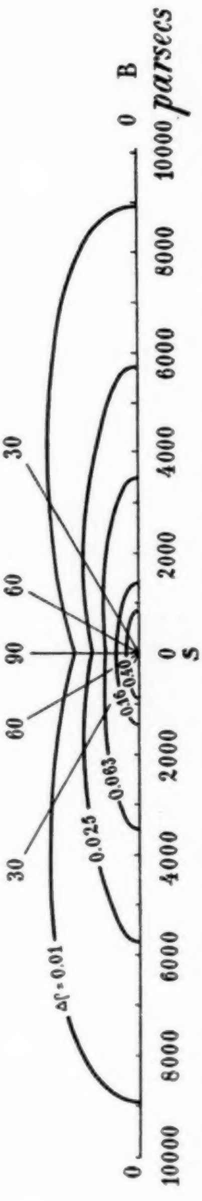


FIG. 2.—Distribution of density in a plane perpendicular to the galactic circle passing through the center of the stellar system. AB is the plane of the galaxy; the marginal numbers 0, 30, 60, and 90 are galactic latitudes. The curves are lines of equal density, the density at the sun (assumed to be at the center of the system) being taken as unity.

[1] By the substitution, below the maximum, of the first solution for the second, the total number of stars N_m which the theory yields exceeds somewhat the total found from observation. But since the volume of space below the maximum of formula (16) is very small, we have assumed that no serious error is introduced.

unity at S, and 0.40, 0.16, 0.063, 0.025, and 0.01, respectively, along the lines shown in the figure.

Whether the inflection near the pole in the lines of small density is real we are not at the moment prepared to say. If it is not, the layers of small density become singularly flat for an enormous range in distance.

We have not yet studied the question of the limits to which our results for the density are fairly reliable; but there is hardly a doubt that we can safely adopt them as a good approximation up to at least 1500 parsecs. In the direction of the pole of the Galaxy this brings us to what many will be inclined to take as practically the limit of the system. At least the density at that distance cannot be 1/200 of that near the sun. In any direction along the plane of the Milky Way, on the contrary, this same limit must be eight times more distant.

Leaving further considerations for the definitive solution, we conclude by expressing the hope that our present attempt to derive the arrangement of the stars will not be misunderstood. We have always considered this problem to be one that must be solved by successive approximations. The first was that attempted in *G.P.* 11, in which the system was considered as a whole, without regard to the differences shown in different galactic latitudes. The present solution constitutes a second approximation. It assumes that the center lies near or in the sun; that the system is symmetrical with respect to the galactic plane, which is supposed to pass through the sun; and that the normal to the plane through the center is an axis of symmetry. In subsequent approximations these assumptions will be dropped. Although perhaps something in the right direction has already been accomplished, much still remains to be done, especially in the study of the galactic clouds, before the solution will be complete.

Astronomical Laboratory, Groningen
January 1920

TEMPORARY SHIFTING ABSORPTION AT THE HEADS OF HELIUM BANDS IN THE SPECTRUM OF GAMMA ARGUS

By C. D. PERRINE

ABSTRACT

Variations in the spectrum of γ *Argus.*—The author has previously called attention to a quasi-periodic change in the Hβ line of this Wolf-Rayet star. Observations continued during 1917–1919 have confirmed this result and led to the discovery of the appearance, during four short periods, of one or two ***temporary, shifting, absorption lines*** on the violet sides of the bright helium bands at $\lambda\lambda$ 3820, 3889, 4026, 4471, and 5016. The shifts of corresponding lines are proportional to the wave-length and indicate absorption by a radially expanding shell of gas whose speed in two cases increased within two or three weeks to a limit of about 1380 km while in another case it decreased somewhat. Besides these measurable absorption lines there were other fainter ones on both the red and violet sides of the bands. During one active period two other helium bands, $\lambda\lambda$ 4650 and 5690, showed absorption lines, but the shifts were constant and relatively greater than those just mentioned. The hydrogen bands showed no absorption lines. These results suggest great activity on the surface of the star.

In a prevous note[1] I called attention to a quasi-periodic change in the Hβ line of this, the brightest of the Wolf-Rayet or Class O stars. Subsequent observations fully confirm those changes and have revealed others, somewhat analogous, in the other hydrogen and in the helium radiations.

Absorption at the heads of the five stronger helium bands has also been disclosed which behaves apparently as does absorption in the novae. Changes in the positions of this absorption form the chief object of the present paper.

γ Argus has been under observation here for nearly two years. During the first half of this year it has been observed on almost every clear night. The spectra have been obtained with the astrographic equatorial provided with a light flint objective-prism of 20° angle and 5 inches aperture. The spectra extend from about λ 3600 to λ 5020, have a dispersion of approximately 33 A per millimeter at Hγ, and are of excellent definition. Some of the recent photographs have been taken on Seed Orthochromatic

[1] *Astrophysical Journal*, **47**, 52, 1918.

plates. None of these however include the λ 5876 line of helium, although they show the Wolf-Rayet band at λ 5690, and an exposure of one hour shows the band at λ 5810. The $H\epsilon$ line of hydrogen and the H line of calcium are clearly separated in spectra of stars with sharp lines. The small aperture has reduced considerably any ill effects of poor seeing so that the varying widths and intensities of absorption lines cannot be ascribed to that cause. The spectra are, of course, without comparison lines from an artificial source.

It was soon seen that the spectrum appeared to be unstable, but the first changes observed, outside of that in the $H\beta$ line, were so evanescent as to make it difficult to decide if all of them were real. The widths of the spectra were then increased, which aided greatly in identifying the faint bands.

The first really important clue to the behavior of the bright bands and absorption lines was obtained from the plate of January 18, 1919, which showed a bright reversal of a line at the position of the H_ζ line of hydrogen. From the fact that not a single instance has been noted among the more than forty bright-line hydrogen stars observed here in which a high line of hydrogen has brightened while lower lines remained dark, it was at once suspected that this activity was in the strong helium radiation near that position.

Plates of March 7 and May 5 showed such strong absorption at the heads of bright bands that it has been possible to identify it with certainty as belonging to the four strongest helium lines in that region of spectrum, and at the head of the intensely bright band at λ4650. Absorption is almost certainly present also at λ4121 but so involved with the hydrogen radiation as to be unsuitable for measurement. It has also been identified at λ 3820, but the spectrum there is not strong enough on most of these plates to permit measurement. Traces of it in other radiations are suspected.

All of the stronger helium emission has been identified. It is all broad and much of it, perhaps all, divided by absorption as in the hydrogen bands. The λ 4686 band at times shows the same characteristic division. It has been possible also to identify several lines of the ζ Puppis series. These do not concern this paper, however, further than to say that two conditions, central absorption and unequal brightening of the two edges, are very

important factors in identification. Variations of intensity and structure have been confirmed in other than the $H\beta$ radiation.

The most striking peculiarity, however, is the appearance, at about the times of transition of the $H\beta$ line, of absorption lines at the heads of the brighter helium bands of emission. In two instances, March 7 and May 24, this absorption at λ 3889 (the strongest in the ordinary photographic region of the spectrum) is clearly double and the others so wide as to indicate duplicity also. This absorption at λ 3889 remained double for several days when one of the components disappeared, leaving but one, presumably the lower.

Practically all of our plates of γ Argus show absorption some 16 or 17 A above the λ 3889 line or at λ 3873$\pm$. The maximum of this absorption shifts about somewhat and its appearance is variable. When there is no absorption visible at the other helium emission, this absorption is hazy and fainter, although usually more conspicuous than that above some of the other helium bands. At the time of the doubling of the absorption above λ 3889, both components are very strong and sharp and very similar in appearance but very different from that at other more normal times. At such times (of doubling) the higher of the components is usually a little nearer the red than when it is hazy and fainter.

The identification of this more or less constant absorption is a matter of some interest. There is a very faint line in the artificial spectrum of helium at λ 3872.[1] Considering the behavior of the rest of the helium radiations in the spectrum of γ Argus it seems doubtful if the absorption at λ 3873$\pm$ can be due to this helium line, however.

An examination of early-type spectra observed here shows that traces of absorption at this point are almost universal in the "bright $H\beta$" stars but rare in others of B type. Harvard observers give a faint line at λ 3872.4 reaching a maximum intensity in the middle and later subdivisions of class B.[2] The same authorities give a line at λ 3872.7 (Rowland) which is strong in the spectra of solar stars but disappears before B-type spectra are reached.

[1] Runge and Paschen, *Astrophysical Journal*, **3**, 8, 1896.

[2] *Annals of Harvard College Observatory*, **28**, 53, 1897.

From this it seems doubtful if there is any connection either with the faint helium line or with the lines found in spectra of later types. Careful measurement and a study of a considerable amount of data will be necessary to decide. These facts raise the question whether this persistent absorption at λ 3873 in γ Argus may be a more or less permanent "shadow" of the helium emission at λ 3889.

Absorption at the head of λ 5016 has been detected in but few of the plates. This is probably due in part to the weakness of most of the spectra in that region. Its definite appearance, however, on March 7 and on May 24, when λ 3889 was doubled, leads to the suspicion that it may be observable only at times of unusual activity or that the intensity of this absorption may decrease with increasing wave-length.

Similar head-absorption has been identified at four epochs including the first plates obtained on August 15, 16, and 17, 1917. The width and strength of the hydrogen central absorption vary considerably. In some cases there appears to be a widening at or before the time that the head-absorption of helium appeared. Displacements of those absorption lines which could be observed with certainty are given in the following table. The displacements are, on the assumption of motion, all negative, of course.

The agreement of the displacements of the helium lines on the assumption of radial motion is as good as can be expected under the circumstances. A rapid increase is also shown.

The discordance and constancy of displacement of the λ 4650 line are noticeable. Whether or not these are real, further investigation alone can determine. No personality or error of measurement can reconcile the discordance if the middle of this bright band is used. The band is, however, weaker on the red side and it may be that it is composite and that the line observed is due to a radiation nearer its upper edge. Color is lent to such a possibility by the anomalies which exist in the radiations in this region in the O and B stars. There is no doubt of the reality of this absorption at λ 4650. A careful examination of practically all of the plates indicates its genuineness. It is curious, however, that no increase in its displacement is shown in ten days. From the fact that there is in some cases a slight brightening at the upper

DISPLACEMENTS OF ABSORPTION LINES IN γ ARGUS WITH CORRESPONDING VELOCITIES OF APPROACH

1919	λ 3889		λ 4026		λ 4471		λ 5016		Mean Helium	λ 465		λ 569	
	A	km	A	km	A	km	A	km	km	A	km	A	km
March 7	15.7	1210											
	13.1	1010	13.7	1021	16.6	1114	18.0	1076	1055				
March 8	17.3	1334											
	13.6	1049	14.5	1080	16.1	1080			1070				
March 15	17.5	1349							1349				
March 16	17.4	1342							1342				
March 18	17.9	1380							1380				
May 2	6.9	532	7.7	574	10.2	684			597				
May 3	11.1	856	8.4	626	12.5	839			774				
May 4	12.2	941	10.5	782	12.1	812			845	21.6	1393		
May 5	12.1	933	11.6	864	14.0	939			912	21.8	1406		
May 6	12.5	964	12.7	946	14.0	939			950	23.1	1490		
May 9	13.8	1064							1064				
May 10	15.4	1187	15.0	1112			20.2	1208	1169	23.1	1490	27.9	1479
May 14	16.7	1288							1288	22.3	1438		
May 24	16.8	1295											
	13.4	1033	12.7	946	15.7	1053			1011				
May 25	16.5	1272	15.0	1118	17.4	1168			1186				
May 26	14.9*	1149	13.4	998	16.8	1127			1090				
May 27	17.4	1342											
	11.7	902	12.3	916	14.0	939			919				
May 28	18.0	1388											
	11.2	864	10.7	797	14.0	939			867				
May 31	17.7	1365											
	10.4	802			12.3	825			814				
June 9	17.9	1380											
	14.8	1141			15.3	1027			1084				

* Double

edge of this absorption line, it is just possible that what is taken to be absorption may be the *central* absorption of some entirely different radiation. If a similar behavior at λ 5690 is verified, however, such an explanation as the latter is rendered improbable. Perhaps the most plausible explanation, in view of the fact that its displacement agrees well with those of the shadow of λ 3889 when that appears stationary, is that this is a real "shadow" of λ 4650 which has reached the stationary stage. A similar displacement of the absorption above λ 5690 is worthy of note in this connection.

No certain traces of absorption have been detected at the heads of the hydrogen bands. In the helium there appears to be a closer relation to the absolute brightness of the bands than the "series" to which the radiations belong.

The most striking feature is the rapid increase which is shown in the displacements of this absorption. These increases are so evident that a simple comparison of the photographs is sufficient to reveal them. They are fully established in three distinct apparitions. A decrease is shown in one.

An increase of the same nature was suspected in our plates of Nova Aquilae No. 3 but could not be readily confirmed on account of the lack of an artificial comparison spectrum and of time for measurement and discussion of the plates. Adams' results, however, appear to fully establish such an increase in that star.[1]

There is evidence in the results for γ Argus that this increase was becoming slower toward the ends of the apparitions of these absorption lines. There are indications of a slowing up also in Adams' results for Nova Aquilae.

Beside the absorption lines at the heads of bright bands which have been measured and are given in the table, there are evidences of others much fainter in the bands themselves, not only in the helium radiations but in hydrogen and others whose origin is as yet unknown. Such absorption lines have been found not only in the violet sides of the bands, but in the red sides also. Those in the red sides are fainter and fewer than in the violet sides.

In a paper on the spectrum of Nova Aquilae No. 3 which was sent to the 1918 meeting of the American Astronomical Society I

[1] *Proceedings National Academy of Science*, **4**, 355, 1918.

referred to the possibility of these displacements being due to motion. Adams in the above-quoted paper in the *Proceedings of the National Academy* also suggests the possibility of motion. F. J. M. Stratton,[1] discussing the absorption spectrum of Nova Geminorum II, March 1912, assumes a Doppler effect as the cause of the similar phenomena in that star also.

The similar behavior of absorption in γ Argus with the advantage of being observed over several and longer intervals tends still farther, in my opinion, to limit the phenomena in question to motion. The wide range of displacements in γ Argus, the slowing up toward the ends of two of the apparitions, the decreasing displacements at one of the apparitions, are what might be expected from more or less radial expansions of an outer shell or shells of gaseous matter, the result of great activity, at least on the surface of the star. Such changes of displacement seem to harmonize better with motion than with purely physical or chemical changes.

The absorption lines on the red sides of the bands may also not be inconsistent with motion, such absorption coming possibly from the portions of the shell which are in general on the opposite side of the star and moving away from the observer, combined with pressures. Such an explanation requires a considerable degree of transparency of such shells of gas which may or may not be the case. So far as my data go, both for γ Argus and for novae, there is less detail and in general less sharpness in the red than in the violet side of the bands. Some such condition might be expected from gaseous shells having a considerable degree of transparency, yet not perfectly so. The desirability is indicated of following such early-type stars as have shown similar absorption bands sufficiently to determine if they may also be subject to changes of position.

Consideration of the relations of these phenomena to the novae and other early-type objects is necessarily reserved until we have more observations not only of this star but of others of its class. It can scarcely be doubted, however, that these changes tend still further to link together the activities of the novae with spectra of early type in which may be included the B stars, particularly those with widened lines and hydrogen emission.

[1] *Monthly Notices R.A.S.*, **79**, 366, 1919.

The results as far as the investigation has progressed may be summarized as follows:

1. Absorption lines have appeared on the violet sides of the bright helium bands in the spectrum of γ Argus at $\lambda\lambda$ 3820, 3889, 4026, 4471, and 5016, lasting for some days.

2. These absorption lines have been observed at four different epochs, August 1917, March 1919, May 1919, May–June 1919.

3. At the epochs beginning March 7 and May 24 at least one of these absorption lines was double and the others perhaps so.

4. The displacements of these absorption lines have increased with the lapse of time in three cases. In the fourth they have decreased.

5. The increase of displacement appears to be less toward the ends of two of the apparitions than at the beginning.

6. This helium absorption appears to be independent of the *series* to which it belongs but to depend upon the intensity of the radiation.

7. The displacement of this head-absorption of helium is proportional to wave-length.

8. On some of the spectrograms dark lines have been observed on the violet sides of the brilliant band at λ 4650 and on one plate, above the band at λ 5690. Those at λ 4650 appear, however, to be stationary, no increase of displacement being observed.

9. All of the stronger helium lines in the portion of the spectrum observed have been identified. This, like the hydrogen, appears to be divided by central absorption.

For the success of this investigation I am greatly indebted to Second Astronomer Winter, whose perseverance in the face of trying weather conditions could alone have secured the observations. A few of the plates were secured through haze and clouds and some of them in the only intervals possible in an entire night. These adverse weather conditions, however, do not affect the validity of the conclusions herein contained. These are fully confirmed by the spectra obtained in a clear sky which are greatly in the majority.

Observatorio Nacional Argentino, Córdoba
October 27, 1919

THE EXTENSION OF THE ULTRA-VIOLET SPECTRUM

By R. A. MILLIKAN

ABSTRACT

Extreme ultra-violet spectra to 202 A.—Using high-potential sparks in a vacuum as sources of light and a high vacuum spectrometer with specially ruled concave gratings, the author has succeeded in extending the spectra of carbon, zinc, iron, silver, and nickel to λλ 360.5, 317.3, 271.6, 260, and 202, respectively. *Vacuum spark spectra with carbon, zinc, iron, and nickel electrodes* are shown in Plates I–III. In all of these spectra there are lines which extend much beyond the width of the spark and are presumably due to gases driven from the electrodes. One of them is probably the hydrogen line λ 1215.7; but though most of the others are identical with the lines obtained by Lyman from his *condensed discharge in helium*, reasons are given for believing that these gas lines are associated with carbon rather than with helium.

Vacuum spark spectra of carbon from λ 7000 to λ 360.—The main lines above λ 2000 were identified and those below λ 2000 are given on Plate III, accurate to within about 0.2 A.

L series of the X-ray spectrum of carbon.—Evidence is presented for believing that the whole spectrum which the carbon atom is able to emit up to and including its X radiations of the so-called L series has now been obtained. Previously no lines of the L series of any element with an atomic number less than 30 had been identified.

High-potential vacuum sparks are sources of X rays, and appear to produce hydrogen.

INTRODUCTION

This preliminary report upon the work which was begun in the Ryerson Laboratory somewhat more than four years ago, with the aid of a grant from the Rumford fund and with the object of extending the ultra-violet spectrum through the use of "hot sparks" in vacuo, was presented to the National Academy of Sciences at the New Haven meeting in November, 1919, but publication has been deferred until the wave-length measurements could be made as precise as possible, and until all of the most important lines involved in the report could be obtained by independent work on different gratings. This new work has been attended by considerable difficulties, the sources of which will be considered in more extended reports, but the results are all essentially the same as those presented at New Haven, and it is thought that all possible question as to the validity of the main conclusions has now been removed.

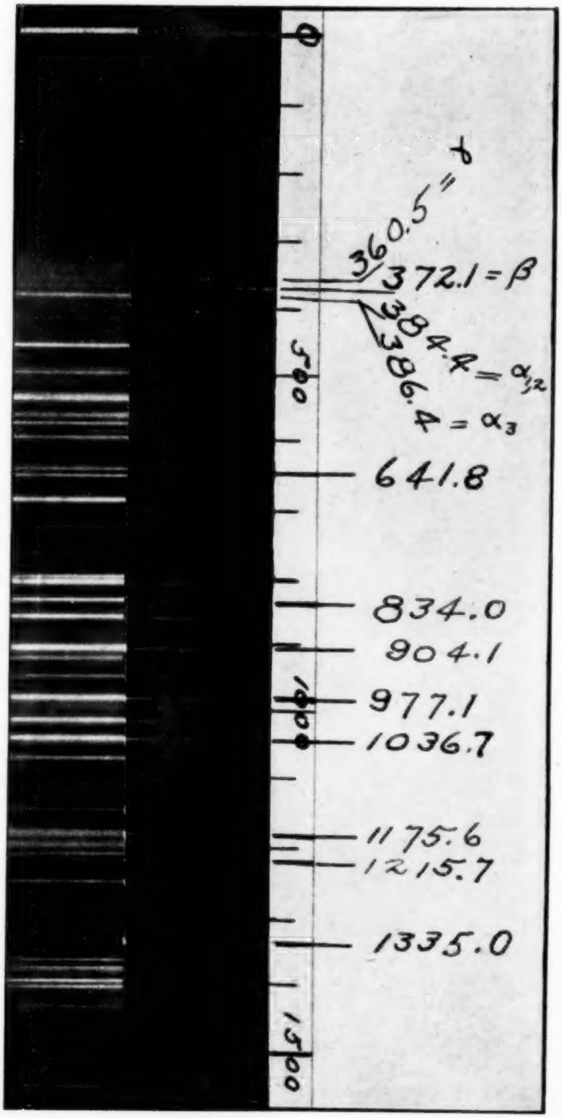
As indicated in previous brief reports made in August, 1918, and August, 1919,[1] the method used grew out of preceding long-continued studies on spark potentials in very high vacua which showed the possibility of removing all limitations imposed (1) by the absorbing properties of residual gases, and (2) by deficiencies in the radiating properties of the source, and of therefore pushing the ultra-violet spectrum up to a point which was limited only by the properties of the grating itself. The long experience of the Ryerson Laboratory in the production of gratings seemed to render this a favorable place for reducing grating limitations to a minimum. Any novelty which the present work possesses lies then (1) in working in an essentially perfect vacuum; (2) in using a source which consists of very high potential sparks between metal electrodes very close together, thus insuring the conditions for the production of the highest frequencies, even of the X-ray type; (3) in producing gratings of somewhat unusual properties, namely, properties which throw as much light of short wave-length as possible into the first-order spectrum, and which have sufficient regularity of ruling to produce good images when the ratio of grating-space to wave-length is as much as 70, instead of from 3 to 6, such as is common practice.

The vacuum spectrometer and accessories used in this undertaking were designed by the writer in the spring of 1916, built in the Ryerson Laboratory during the summer of that year, and experimented upon by him throughout the succeeding fall; but in the winter, being called to service in connection with the war, he left the immediate conduct of the work in the hands of Mr. R. A. Sawyer, who had been assisting him. Before the latter was called to the service in the following spring, some thirty new zinc lines between $\lambda\lambda$ 2000 and 900 had been brought to light. Since these experiments of necessity had to be discontinued, these preliminary results were published (*op. cit.*), although they had thus far been disappointing in the main object sought, namely, the extension of the ultra-violet spectrum.

Immediately after his release from the service in January, 1919, however, the writer went at the problem again and introduced

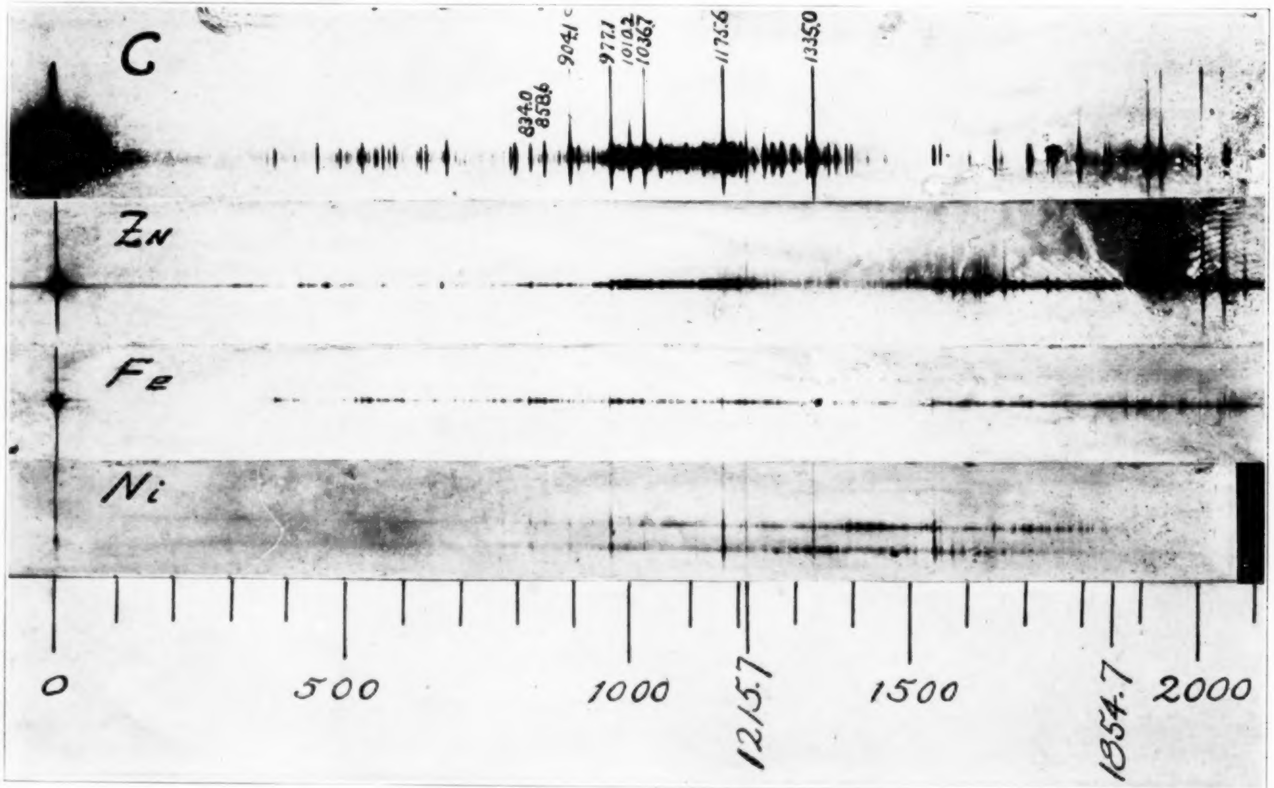
[1] Millikan and Sawyer, *Physical Review*, **12**, 168, 1918; and *Science*, **19**, 138, 1919.

PLATE I



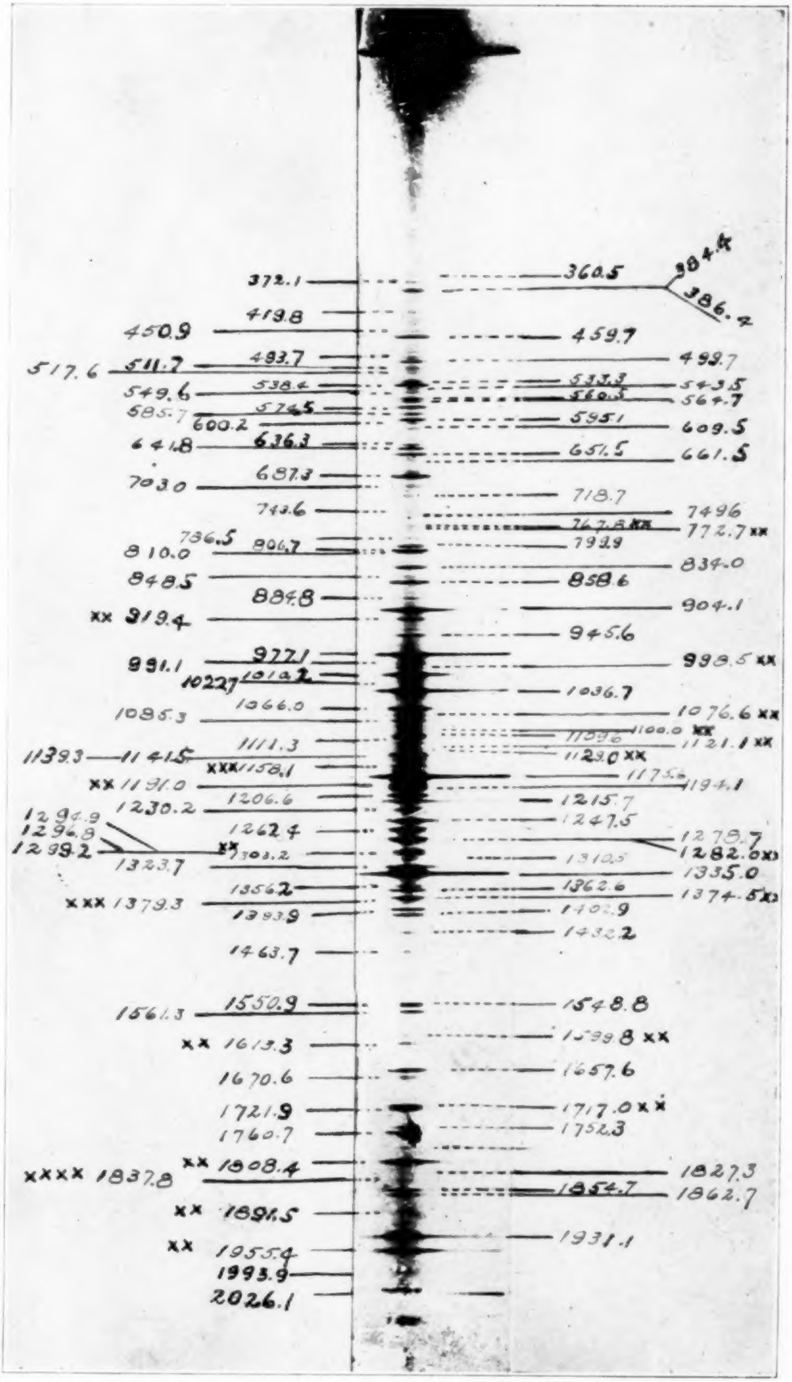
CARBON SPECTRUM BETWEEN λ 1500 AND λ 360.5

PLATE II



EXTREME ULTRA-VIOLET SPECTRA OF CARBON, ZINC, IRON, AND NICKEL

PLATE III


372.1
360.5
384.4
386.4
419.8
450.9
459.7
517.6
511.7
493.7
499.7
538.4
549.6
585.7
600.2
609.5
641.8
636.3
651.5
661.5
703.0
687.3
718.7
743.6
749.6
786.5
810.0
806.7
834.0
848.5
858.6
884.8
904.1
919.4
945.6
991.1
977.1
1036.7
1085.3
1066.0
1076.6
1111.3
1139.3
1141.5
1158.1
1129.0
1191.0
1194.1
1230.2
1206.6
1215.7
1247.5
1294.9
1296.8
1299.2
1262.4
1303.2
1323.7
1335.0
1356.2
1362.6
1379.3
1374.5
1393.9
1432.2
1463.7
1561.3
1550.9
1548.8
1613.3
1670.6
1657.6
1721.9
1717.0
1760.7
1752.3
1837.8
1808.4
1827.3
1854.7
1862.7
1891.5
1955.4
1931.1
1993.9
2026.1

improvements both in the vacuum and in the grating which, with the assistance of Mr. Sawyer, who returned from the service in May, 1919, produced the desired results, and in August, 1919 (*op. cit.*), a brief report was made of the extension of the ultra-violet spectrum to 320 angstroms.

The objects of the present paper are:

1. *To report that still further work has carried the ultra-violet spectrum to 202 angstroms.*

2. *To present and to discuss briefly a few of the most striking of the photographs already obtained.*

3. *To present evidence that the characteristic L series of X rays of carbon have now actually been obtained by ordinary mechanical gratings and that thus the gap between X rays and light appears to have been closed up.*

I. THE LIMITS THUS FAR REACHED WITH DIFFERENT SUBSTANCES, AND THE METHODS EMPLOYED TO REACH THEM

The spectra herewith presented were obtained by intermittent sparking between electrodes from 0.1 mm to 2 mm apart with a battery of leyden jars (navy jars) charged to potentials of several hundred thousand volts by a powerful induction coil. The gases evolved by the sparking render the attainment of this type of spark impossible after the pressure has risen above about 10^{-4} mm. Hence a mercury diffusion-pump attached to the vacuum spectrometer was kept in continuous operation and the period between sparks rendered long enough to maintain the pressure at somewhat less than the foregoing value. Although it takes at least a day to obtain a single plate the actual time of sparking per plate is in general not over thirty minutes.

Some eight different gratings have been ruled and used and the main lines checked upon several of them which possess different grating constants varying from 500 to 1100 lines per millimeter, so that the peculiarities of individual gratings have been altogether eliminated. However, it is clear that with a ratio of grating-space to wave-length of as much as 70 in the case of the shortest wave-lengths obtained, the differences in the performance of different gratings are bound to be very marked. Although, then, all of the

gratings show the same main lines in the region below 1000 angstroms, two of them have given spectra which are greatly superior in distinctness and in definition to those obtained with the others, and the photographs here shown were obtained with one of these two gratings. Its focal length is 83.5 cm and its constant 505.3 lines per mm. The spectrometer has been sufficiently described in preceding reports.

The substances of greatest interest for studies of this kind are clearly those of small atomic number, for no X-ray spectra of the L series have ever been taken with crystal gratings in the case of elements of atomic number lower than 30. Furthermore, substances of atomic number much lower than this are forever beyond the range of the methods of X-ray spectrometry, because of the fact that the wave-lengths of the L rays from such substances become so large in comparison with the grating-space of crystal gratings that sharp images can no longer be formed. By far the most interesting spectrum, then, which has yet been taken is that corresponding to the substance of smallest atomic number which has been thus far successfully used, namely, carbon, atomic number 6. Furthermore, it so happens that carbon enables us to use the largest electrode distance, namely 1 mm and more. The carbon spectrum will be considered somewhat at length in section iv. The shortest wave-length which it shows is at 360.5 angstroms. The limitation is not then imposed by the grating, however, for the same grating gives spectra with both zinc and iron which run down to considerably shorter wave-lengths.

The shortest wave-length from zinc which has thus far been obtained lies at 317.3 A; that from iron lies at 271.6 A, that from silver at 260 A, while *the lowest limit thus far reached at all is obtained with nickel, and has a value 202 A*, between one and two octaves farther down than the lowest values previously attained.

II. COMPARISON OF SPECTRA OF DIFFERENT ELEMENTS

Plate I shows the spectrum of carbon, the upper portion being an unmodified reproduction of the plate itself, while the lower portion represents the same spectrum after it has been stretched vertically by a process similar to that usually used for stretching

stellar spectra. The plate from which this lower spectrum is made has also been retouched in the region of the central image seen to the left, but it has not been touched at all in the region in which the lines appear.

It will be seen that in place of the very small number of reproducible lines which any substances have heretofore yielded below 1000 angstroms,[1] the spectrum obtained in this way by sparking between electrodes about 1 mm apart made from battery-jar carbon is a very strong and rich one containing more than 50 clearly reproducible lines.

Plate II shows photographs which have been obtained with electrodes of carbon, zinc, iron, and nickel. For convenience of study these four spectra have been reproduced as nearly as possible on the same scale. Plate III shows again the carbon spectrum with all of the most important wave-lengths inserted. These represent entirely new determinations based on the aluminum lines $\lambda\lambda$ 1854.7 and 1862.7 and should be correct in the case of the distinct lines to within one- or two-tenths of an angstrom.

It will be seen that the carbon spectrum is much the most brilliant and much the most simple of those presented on Plate II. Indeed, on account of the very small distance at which the sparks can be forced across in vacuo between electrodes of most of these metals (in some cases but a few tenths of a mm with potentials of at least 200,000 volts), the reproductions are much less perfect than in the case of carbon. The shortest wave-length which is clearly visible on the plate in the case of iron probably does not show in the reproduction. It is hoped, however, that the reproduction will show the iron line at 290.7 angstroms.

It will be seen that in all the spectra there are lines which extend beyond the limits of the spark and are visible throughout most of the length of the slit, in some cases throughout its whole length. These are presumably due to gases which are driven out of the electrodes by the spark and which form a faint aureole about them. I would call attention first to the remarkable fact

[1] The only reproductions which have ever been shown in this region are found opposite page 89, *Astrophysical Journal*, **43**, 1916. Cf. also *Proceedings Royal Society*, A, **95**, Plate 3, though no lines below λ 1000 can here be seen.

that the most prominent of these "gas lines" are clearly discernible in all the plates. One of them, namely, the line marked 1215.7, is the longest wave-length in the so-called Lyman series of hydrogen. It appears conspicuously in all the spectra, though it is not nearly so intense as some of the other "gas lines." No other of this Lyman series of hydrogen lines is sufficiently intense to show on these plates. Furthermore, this 1215.7 line is easily distinguishable on all the plates from the other lines by the fact that it shows almost no broadening whatever in the center, while many of the other lines show quite marked broadening, which must be due to the Stark effect, or to the pressure effect, or to both combined. According to Paschen's exceedingly careful determination[1] of the Rydberg constant for hydrogen, viz., $N=109677.659$, the correct position of this line, if it is due to hydrogen, should be at 1215.676 A, that is, *exactly where it is here found*. The corresponding helium line (using $N=109722.144$) should be at 1215.2. It is perhaps possible that the present accuracy of our measurements would not permit us to distinguish with certainty between these two lines, although if both were present the line on the plate should be less beautifully narrow and well defined than is the case; but in any event the presence of this line in all of our spectra appears to demonstrate without a reasonable doubt that *either hydrogen or helium, or both, are evolved from all of our electrodes by the sparking process*. Nevertheless, although we have taken photographs with a Hilger quartz spectrometer and panchromatic plates of the whole spectrum of these sparks from λ 2000 up to λ 7000, I have thus far been unable to find any traces of the characteristic hydrogen or helium spectra in this region of longer wave-length. The evidence furnished, however, by λ 1215.7, appearing as it does in all our spectra, is so convincing that I am disposed to consider the failure in the visible region to be due to lack of sufficient intensity. Further evidence on this point is now being diligently sought. The only other lines appearing on our plates which should be associated either with the hydrogen line λ 1215.7, namely, λλ 1025.7 and 972.5, or with the theoretical helium line at λ 1215.2, namely, λλ 1640.5, 1085.0, 1025.3, etc., is a line at λ 1085.3 which

[1] *Annalen der Physik*, **50**, 935, 1916.

has about half the intensity of λ 1215.7 and which has very nearly the wave-length corresponding to $m=5$ in the helium series $4N\left(\frac{1}{2^2}-\frac{1}{m^2}\right)$, N being 109722.144. The absence of all of the other lines of this series, some of which should be more intense than this one, would seem to indicate that this may be an accidental agreement.

The second remarkable fact revealed by a study of these spectra is the following: The other strong gas lines which are common to all the plates will be seen from a glance at Plate II to be $\lambda\lambda$ 1335.0, 1175.6, 1036.7, 1010.2, 977.1, 904.1, 858.6, and 834.0. *Now all save two of these are identical with the strong lines which Lyman obtained from his condensed discharges in helium at from 2 to 3 mm pressure.*[1] This will be clearly seen from the following table taken in connection with a study of the wave-lengths recorded on Plates I, II, and III. The first column of the table contains the wave-lengths of the whole group of lines reported by Lyman.[2] The second column represents Lyman's estimates of the intensities of these lines. The third column represents the corresponding wave-lengths as measured by one of my assistants, Mr. G. D. Shallenberger, on a plate taken with a grating having a focal length of 83.5 cm and a grating constant of 505.3 lines per mm, while column 5 gives the wave-length as measured by a second assistant, Mr. Ira Bowen, on a plate taken nine months later, with another grating of focal length 83 cm and a grating constant of 564.7 lines per mm. Columns 4 and 6 give the intensities of the lines as they appear on these two plates according to the estimates of Mr. Shallenberger and Mr. Bowen, respectively. Column 7 gives the final corrected values of these wave-lengths as obtained by the method sketched in the next section. These measurements were made upon the spectrum of carbon. The readings of two observers are introduced merely for the sake of showing the sort

[1] *Astrophysical Journal*, **43**, 102, 1916.

[2] Since the width of slit used in taking the plate here under consideration made it difficult, if not impossible, to distinguish weak lines lying closer together than about 1.5 A (dispersion 23 A per mm), I have in column 1 modified Lyman's table to the extent of recording merely the center of his close doubles, rather than the components.

of agreement obtained with different gratings measured by different observers.

TABLE I

COMPARISON OF SPECTRA OBTAINED FROM HELIUM AND FROM CARBON

Lyman Helium	Estimated Intensity	Carbon No. 1	Estimated Intensity	Carbon No. 2	Estimated Intensity	Final Corrected Values
599.0......	1	600.6	1	600.0	0	600.2
643.7......	1	....	....	....	....	....
702.9......	3	703.1	2	703.3	1	703.0
718.2......	5	718.8	3	719.0	1	718.7
796.8......	2	....	....	....	....	....
834.1......	8	834.15	5	834.3	6	834.0
904.6......	4	904.3	10	904.5	8	904.1
916.7......	2	....	....	917.1	1	....
972.7......	1	....	....	....	....	....
977.2......	6	977.3	11	977.5	10	977.1
991.1......	3	991.2	3	991.5	2	991.1
1010.6......	4	1010.5	11	1010.5	9	1010.2
1026.0......	4	....	....	....	....	....
1037.0......	5	1036.9	12	1037.0	10	1036.7
1085.5......	5	1085.5	4	1085.4	3	1085.3
1134.7......	1	....	....	....	....	....
1175.9......	10	1175.7	13	1176.5	10	1175.6
1199.8......	5	....	....	....	....	....
1216.0......	10	1215.8	7	1216.0	5	1215.7
1236.0......	1	....	....	....	....	....
1247.9......	5	1247.8	10	1247.8	7	1247.5

Perhaps no especial significance is to be attached to the approximate coincidence between Lyman's and our own measurements upon the first three lines in the table, for these are faint lines in our plates surrounded by strong ones, as a glance at the reproductions will show. Our wave-lengths $\lambda\lambda$ 600.2 and 718.7 are respectively 1.2 A and 0.5 A higher than Lyman's and this approximate coincidence may be accidental, for the divergence exceeds the errors in our measurement even in the case of these imperfect lines. The coincidence at λ 703.0, however, is not likely to be accidental, and beginning with λ 834.0 *every one of Lyman's strong lines save his hydrogen line 1026.0 already discussed is also a strong line in our spectra* and most of his lines are those which in our plates will be seen to extend the full length of the slit. The coincidence is remarkably exact, as a glance at the first and last columns of Table I will show. The differences are quite uniformly 0.3 A,

which indicates that they are due in the main to differences in the determination of the grating constants. The comparison also indicates that Lyman's measurements are much more exact than he claims. He allows a possible uncertainty of an angstrom. It will be seen from Table I that there are but five lines weak or strong given by Lyman in the region between λλ 800 and 1250 which do not appear in our carbon, and two of these five, namely, λλ 1026.0 and 972.7, belong to the hydrogen series which begins theoretically with λ 1215.7 and converges at λ 911.76.

I at first thought that the appearance on these plates of practically the whole series of lines which Lyman obtained from helium was strong evidence that we were producing helium through the disintegration of the elements of our electrodes. But the non-appearance in the spectra of these hot sparks taken with a quartz spectrometer in the region between λλ 2100 and 7000 of any trace of helium lines and the appearance on these same plates of a whole series of strong "gas lines" which are clearly due to carbon is favorable to the hypothesis that the element which is responsible for this series of strong gas lines on these plates and on Lyman's plate also is *carbon*. Carbon might be expected to be an impurity in all of the metals used in these experiments, and it would almost certainly be an impurity in Lyman's experiments.

However, we have recently taken a hot-spark spectrum using *chemically pure silver electrodes and though this substance yields 600 lines between 260 angstroms and 2000 angstroms, none of the lines in question are present, save a faint one at 1215.7 and another at 976.9. This is very strong evidence in favor of the carbon hypothesis.* The only evidence against it is found in the fact that there is a complete lack of agreement between the carbon spectrum revealed in our plates and the CO spectrum as given by Lyman and other observers. The CO spectrum is, however, widely different in other regions from the carbon spectrum. Further, some of the recent work on atomic structure would indicate that the CO molecule might be expected to furnish none of the characteristic lines of the carbon atom itself. *Of the two hypotheses, then, which are at present possible, namely, that the strong group of gas lines which appear in all of the present spectra and which also appeared in Lyman's work on helium*

are due either to the helium atom or to the carbon atom, I think the present evidence is strongly in favor of the latter hypothesis.

III. THE RELIABILITY OF THE PRESENT WAVE-LENGTH MEASUREMENTS

As to the accuracy of the wave-lengths shown in this table and on the plates, they are obtained in each case by using the aluminum lines λλ 1854.7 and 1862.7 as standards. These lines were found to be present in the carbon spectrum and they were obtained independently by using aluminum electrodes as a source. The wave-lengths here used are so short and the incidence so nearly normal that the spectrum itself is practically a normal one. The largest correction which it was found necessary to apply to any wave-length between zero and λ 1854.7 to allow for the departure from a linear relation between wave-length and displacement from the central image was found by careful measurement of angles and by geometrical analysis to be but 1.6 A. The proper corrections have, however, been made in every case, so that columns 3 and 5, Table I, represent new determinations of absolute wave-length, save as they depend upon λλ 1854.7 and 1862.7 taken as standards. The values assigned to the standards represent Runge's[1] measurement in air, viz., 1854.09 A, reduced to vacuum, 1854.77 A, and then reduced as well as can now be done from the Rowland system to the International system adopted by the Solar Union.[2] It is the value adopted by Lyman, and there seems to be no doubt that it is correct to a tenth angstrom. The differences between columns 3 and 5, Table I, show that the uncertainty in any of our individual wave-length measurements should not be more than a very few tenths of an angstrom at the most.

But there exists a still more reliable check upon the accuracy of these determinations of wave-length than that found in their consistency among themselves or in their closeness to Lyman's results, namely, this: There is a considerable group of lines in Plate III which represents the second, third, or fourth order of

[1] *Annalen der Physik*, **55**, 47, 1895.

[2] *Astrophysical Journal*, **32**, 215, 1910; and *Smithsonian Physical Tables*, 6th ed., 1914, p. 173. See also Kayser's *Handbuch der Spectroscopie*, Vol. VI, p. 1033.

lines which are found in the first order nearer the image of the slit. The second-order lines in Plate III are marked by two crosses, the third order by three crosses, and the fourth order by four. A most conspicuous example of this appearance of lines in higher orders can be seen by a glance at Plate III. The lines λλ 1599.8 and 1613.3 are quite clearly the second order of the pair of strong lines at λλ 799.9 and 806.7. They show diminished intensity and have twice the dispersion of the first-order lines, just as they should. The completeness of this type of evidence is shown in Table II, in which are collected the twenty-three lines appearing in Plate III which are second or higher orders of other lines found on this plate. The intensities of these lines had all been estimated before the discovery of their relationship to other lines, but a glance at these intensities in column 2 shows that they all bear as nearly the same ratio to their respective originals in column 4 as could be expected from estimates of this kind.

Now it is clear that the existence of these lines of higher order makes it possible to make the wave-length determinations completely independent of any scheme of correction of the measurement of displacements from the central image. For a distinct and exactly measurable second-order line close to the standard λ 1854.7, for example, λ 1808.2, is known with practically the same accuracy as the standard itself, and this gives at once the correct value of the corresponding first-order line as 904.1 instead of 904.3, its value according to the measurements in the third column of Table I. But again, close to this line at 904.1, there is (see Table II) the very sharp and exactly measurable line λ 919.4, which is in turn the second order of one of the best short-wave-length lines, viz., λ 459.7. Thus by going over this whole series of lines which appear in different orders, and assigning weights proportional to their sharpness and ease of measurement, it is possible to plot a final correction-curve to the series of wave-lengths first determined by measurement of angles and geometrical analysis and thus to free the results completely from dependence upon the imperfections in those measurements. The last column of Table II represents the *weights* assigned by Mr. Bowen to the various pairs of lines entering into this correction-curve, and the

last column of Table I, and the plates, then give the final values of the wave-lengths thus determined. It is a most interesting and fortunate fact which appears from Table II that one of the strongest and most accurately measurable of the lines of very short wave-length, namely, λ 459.7, actually appears on Plate III in the first, second, third, and fourth orders, and the last order falls at λ 1837.8, exceedingly close to the standard line λ 1854.7, so that this line alone might suffice for the complete correction of the whole series of wave-lengths.

TABLE II

LINES APPEARING IN MORE THAN ONE ORDER

Lines of Higher Order	Intensity	Order	Corresponding First-Order Lines	Intensity	Weight Assigned to the Pair
767.8	(1)	Second order of	384.4	(4)	(1)
772.7	(1)	Second order of	386.4	(4)	(2)
919.4	(4)	Second order of	459.7	(9)	(10)
998.5	(3)	Second order of	499.7	(6)	(1)
1076.6	(4)	Second order of	538.4	(7)	(3)
1100.0	(1)	Second order of	549.6	(3)	(1)
1121.1	(2)	Second order of	560.5	(4)	(1)
1129.0	(1)	Second order of	564.7	(3)	(1)
1148.2	(2)	Second order of	574.5	(6)	(2)
1158.1	(3)	Third order of	386.4	(4)	(1)
1191.0	(3)	Second order of	595.1	(5)	(1)
1282.0	(3)	Second order of	641.8	(5)	(1)
1303.2	(4)	Second order of	651.5	(6)	(1)
1374.5	(5)	Second order of	687.3	(7)	(12)
1379.3	(3)	Third order of	459.7	(7)	(10)
1599.8	(1)	Second order of	799.9	(5)	(4)
1613.3	(2)	Second order of	806.7	(6)	(8)
1620.0	(2)	Second order of	810.0	(5)	(1)
1717.0	(3)	Second order of	858.6	(5)	(12)
1808.4	(7)	Second order of	904.1	(10)	(10)
1837.8	(1)	Fourth order of	459.7	(7)	(1)
1891.5	(1)	Second order of	945.6	(4)	(1)
1955.4	(7)	Second order of	977.1	(11)	(8)

There are, then, no errors left in our wave-length measurements except the observer's errors in setting upon the center of a given line. This can scarcely be more than a tenth of an angstrom in the case of a well-defined line. It might conceivably reach two- or three-tenths in the case of a weak line. A satisfactory indication of the reliability of these final results is found in the fact that we had fixed the correct position of the first line of the Lyman series

upon our plates at λ 1215.7 before I had computed the theoretical position of this line from Paschen's exceedingly careful measurements of 1916. *The theoretical and observed positions were then found to coincide exactly.*

IV. THE CARBON SPECTRUM

The carbon spectrum differentiates itself sharply from all of the other metallic spectra thus far obtained in this work, first, by the smallness in the number of its lines; second, by their character. Most of the other elements show a great number of lines which are for the most part weak and somewhat diffuse, and which appear (see Plate II) only in the region between the electrodes. In more complete papers which are to follow, the wave-lengths found in these spectra will be fully reported and catalogued. In contrast with this the lines of carbon cover, as previously indicated, a much larger portion of the slit—in some cases the whole of it; they are relatively few in number and exceedingly well defined, at least at the top and bottom, and they seem to be grouped to a considerable extent into a relatively small region of wave-lengths. It will be seen that when the lines of higher orders are omitted the carbon spectrum is considerably more simple than is indicated by a first glance at Plate I. So far as I know, it has never been seen before below about λ 1000. There is nothing with which to compare it except the "line spectrum of carbon" between λλ 1464.5 and 2000, as obtained by McLennan, Ainslie, and Fuller[1] with a prism

[1] *Proceedings Royal Society*, **95**, 327, 1919. There is also a paper by McLennan and Lang in *Proceedings Royal Society*, **95**, 272, 1919, in which plates are shown of the "Carbon Vacuum-Arc Spectrum." Reliable comparison with this is, however, impossible on account of uncertainties in wave-lengths. Their reproduction shows just five beautifully strong lines between λλ 2000 and 1300, the wave-lengths of which the authors place at λλ 1933, 1667, 1574, 1348, and 1343. These ought to correspond to our strong lines λλ 1931.1, 1657.6, 1561.3, 1335.0, and 1323.7 (see Plate III), though it will be seen that some of the wave-lengths differ from ours by as much as 15 angstroms. They also differ by as much as 13 angstroms from the foregoing measurements by McLennan, Ainslie, and Fuller which agree with ours to within 2 or 3 angstroms. It is to be remembered that McLennan and Lang's source is quite distinct from ours, being a vacuum arc, rather than a high-potential spark. This is a possible cause of the differences. Their reproduction shows no strong line below about λ 1300, and but four very faint ones at λλ 1200, 1064, 1039, and 1004, respectively. The authors also state that 3 more are discernible on the plate at λλ 934, 918, and 584. None of these lines below 1200 are with certainty identifiable with lines of our carbon spectrum, though more precise measurements may later reveal such identity.

spectrometer using fluorite lenses and prisms. Their resolution is greater than that here used, but their wave-lengths are presumably not so accurately determinable. They of course obtain none of the second-order lines shown in Plate III. Treating two of their complex lines as singles, in view of the fact that our larger slit width and smaller resolution might not differentiate them, I find that these authors obtain only 9 carbon lines between $\lambda\lambda$ 2000 and 1400, *and these are all, save one, in the present spectrum with approximately the same relative intensities.* They are (see Plate III) $\lambda\lambda$ 1932.1, 1760.7, 1657.5, 1561.3, 1550.8, 1549.4, 1482.2, and 1463.6. But we have also taken, through a quartz window, "hot-spark" spectra of carbon with the quartz spectrometer between $\lambda\lambda$ 2100 and 7000, and we find appearing as "gas lines" the few strong lines in this region which Kayser says are universally recognized as due to carbon itself. These strong lines are $\lambda\lambda$ 2297, 2478, 2512, 2747, 2836, 2995, 3922, 4267, 5144, and 6595. Line 2297 is particularly significant. It is the strongest line on our plate and the only one between $\lambda\lambda$ 2478 and 2000. Eder[1] also reports only this line in this region and places it at λ 2296.85. *These facts prove that carbon itself is able to volatilize in our sparks and to produce the strong "gas lines" which are seen in our plates.* There are some of our weak lines which seem to be due to oxygen, some others probably due to other impurities, though none so far as we can discover due to nitrogen or argon. The evidence then is strong that the main lines exhibited in Plate III constitute the chief radiations from the carbon atom from λ 2000 down. We hope, however, soon to be able to present the spectrum of carbon taken with very pure carbon electrodes and freed from all effects of other substances.

There is already, however, quite convincing evidence that our plates exhibit the whole spectrum which the carbon atom is able to emit *up to and including its X radiations of the so-called L type.* This evidence may be summarized under the following heads.

1. *The carbon spectrum, although by far the most intense one yet taken, stops suddenly at λ 360.5 before the limit of the grating has been reached.* This is shown clearly by the appearance of dozens

[1] *Zeitschrift für wissenschaftliche Photographie* **13**, 33, 1914.

of lines in the zinc, iron, and silver spectra below the carbon line λ 360.5. Further, the zinc and iron spectra were taken with the same grating under the same conditions. This is exactly what is to be expected if λ 360.5 represents the end on the short wave-length side of the L spectrum of carbon. There should be no further line until the K spectrum is reached, whose wave-lengths should be of the order of a ninth of those of the L lines.

2. *According to such extrapolation methods as are now available the L spectrum of carbon ought to appear at just about the point at which the carbon lines do begin to appear in these plates.* It is true that it is not possible to extrapolate with any certainty from Moseley's or Siegbahn's measurements on the L rays of the elements from atomic number 30 to atomic number 90 and obtain any reliable conclusion as to the position of the L lines for substances of as small atomic number as carbon. Uhler[1] has developed extrapolation formulae which I have attempted to use but which show such inconsistencies in this region of atomic numbers that they are clearly inadequate. For example, Uhler's formulae for the L_β line would indicate that its wave-length becomes greater than that of the L_α line at atomic number 6. This kind of inconsistency is also shown by the extrapolation formulae developed by others.

I have accordingly discarded such formulae and have attempted to locate the most probable position of the L_α line for carbon as follows: The measurements on the K series of X rays have been extended from atomic number 92 down to atomic number 11, and the simple Moseley law holds so closely throughout this whole range that the extension of the line from atomic number 11 down to atomic number 6 can scarcely yield a result which is largely in error. I find by this small extrapolation in the K series that the wave-length of the alpha line of the K series of carbon ought to be very close to 45 A. Now, the ratio between the wave-length of L_β and K_α for the element of atomic number 60 is, according to Siegbahn's measurements, 6.5. The ratio between the wave-lengths of the corresponding lines for the element of atomic number 33 is 8.06, and the increase in this ratio between atomic numbers

[1] *Physical Review*, **9**, 325, 1917.

60 and 33 is uniform between these limits. Assuming that this rate of increase continues down to atomic number 6, I find that the beta line of the L series of carbon ought to be 9.6 times the wave-length of the K_α line of carbon. This would place the position of the L_β line of carbon at $45 \times 9.6 = 432$ A. In other words, according to this method, the L spectrum of carbon ought to terminate on the short wave-length side at about 430 A. It does terminate according to these plates at 360.5 A. The fact that the observed value is 70 A *lower* rather than higher than the computed value is favorable to the view that λ 360.5 is the shortest wave-length of the L series of carbon. If the expected position had been below the observed, the evidence would have been the other way.

Again the Kossel law $K_\beta - K_\alpha = L_\alpha$ when applied to the lightest element for which K_β and K_α have both been measured, viz., Mg atomic number 12, gives $L_\alpha = 213$ A. If then the Moseley law held in this region, L_α for carbon would be 4×213 A. But the Moseley law should not hold here, since, for atomic numbers less than 10, the second ring, i.e., the L ring, contains less than eight electrons. For neutral carbon it contains but four; so that the radius of this ring or "shell" should be *considerably smaller* than in the case of atoms of atomic number greater than 10, and hence *the frequency corresponding to the entrance of an electron into this L ring should be considerably greater than that computed by the combination principle and the Moseley law, just as we find it here to be.*

There is a third mode of extrapolation which has been suggested by F. Kirchhof,[1] and which he finds to work well in the region in which the L rays have been measured. According to this author, L_α is found to be inversely proportional to the square of half the atomic weight. If this rule should be applicable to substances of small atomic weight, it would give for the value of L_α of carbon 355 A.

Although, then, these modes of extrapolation are not at all reliable, they do indicate strongly that the L rays of carbon should lie in the general region of wave-length at which the carbon spectrum ends in these plates.

[1] *Physikalische Zeitschrift*, **20**, 213, 1919.

3. *The relative wave-lengths and intensities of the three shortest lines on the present plate are closely similar to those shown by the three main lines of the L series.* It is perhaps not to be expected on theoretical grounds that the L rays of carbon would be *like* those of substances of atomic number more than 10; for the reduction to four of the electrons of the L ring might not merely increase the frequency but it might also cause changes in *relative* wave-lengths. It is interesting therefore that our three shortest lines do show such strong similarity to the three main L lines.

4. *The alpha line of the L series is a double in all elements. Similarly, the strong line which according to this hypothesis corresponds to L_α in carbon is clearly a double* (see Plate III). Further, the ratio between the wave-lengths of the components of this double and the difference in wave-length between the L_α and L_β is approximately what is to be expected from the general relationships found in the L series.

5. Not only *ought* our high-potential sparks produce all the X rays of which the carbon atom is capable, but *we actually find by examining through our quartz window the radiation of these high-potential sparks with a fluoroscope that strong X rays are being given off.* The L series of carbon is then assuredly present in the radiations from our "hot sparks." There is no place for it except at the very end of the short wave-length of the accompanying spectra.

The evidence, then, is extremely convincing that the present plates reveal all of the high-frequency radiations which the carbon atom is able to emit except the K rays of carbon, which have wave-lengths of about 45 A and lie beyond the capacity of the gratings which we are at present able to produce. It will be interesting if we can obtain this same spectrum by direct bombardment with cathode rays unaccompanied by a disruptive discharge of the kind which we are using. Nevertheless, since such bombardment is actually occurring in the production of these hot sparks, the evidence will even then be only slightly more convincing than it is at present. Further, it is to be remembered that our spectra are produced by *free, radiating* carbon atoms, i.e., by

carbon gas, in which electrons can freely enter and leave, without disturbance from outside forces, this second or L ring, whereas in solid carbon the four electrons of the second ring may well form a portion of a space-lattice structure, *which is of such a character as to prevent altogether the generation of these L-ring rays.*

It is perhaps of interest in closing this brief preliminary report to reproduce a diagram taken from Professor Lyman's book

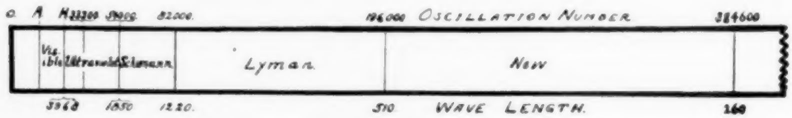


FIG. 1

(p. 105) but modified so as to accord with these new measurements in the ultra-violet. In this diagram the extensions of the various regions of the spectrum thus far explored are presented on a scale which is proportional to frequency $(1/\lambda)$. It will be seen that the extension in frequency shown in Figure 1 is $384,000-196,000$ waves per centimeter, which is more than six times the number of waves added by Schumann's work, namely, $82,000-54,000=28,000$ waves per cm. Since the diagram was prepared a fine new plate which extends the spectrum of Ni to 202 A has been obtained.

This investigation was undertaken with the aid of a grant from the Rumford Fund of the American Academy of Arts and Sciences. I have been ably assisted in it in succession by Dr. R. A. Sawyer, Mr. G. D. Shallenberger, and Mr. Ira Bowen, in collaboration with whom I expect soon to publish complete tables of wave-lengths in this hitherto-unexplored region. Messrs. Sawyer and Shallenberger have done most of the immediate work involved in the taking of the plates, the former assisting up to September, 1919, the latter since then, while Mr. Bowen has been responsible for much of the computation of wave-lengths. The undertaking itself would have been impossible but for the opportunity to utilize the facilities for the ruling of gratings developed by Professor Michelson at the Ryerson Laboratory and but for the skill developed by Mr. Fred Pearson, who has produced all the gratings used.

RYERSON PHYSICAL LABORATORY
UNIVERSITY OF CHICAGO
July 19, 1920